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A model of monopoly with strategic government intervention

T.C.A. Anant, Kaushik Basu*, Badal Mukherji[†]

Delhi School of Economics, University of Delhi, Delhi 110007, India

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Abstract

A formal model of firm–government interaction is developed in which the firm chooses the producer price and the government chooses the ad valorem tax rate to maximise revenue collection. This game is then embedded as the second stage of a two-stage game where, in stage 1, the firm gets to choose its technology and therefore its cost function. The main results of the paper are (i) to characterise the Nash and Stackelberg equilibria of the government-firm game (ii) to demonstrate that 'strategic' inefficiency, i.e. the choice of high-cost technologies when lower-cost options are available, is pervasive in these models whenever the government uses ad valorem taxes, and (iii) to show that, if the government's choice between ad valorem tax and specific or per-unit tax is endogenised, then in a perfect equilibrium the government *will* in fact choose the ad valorem system.

Key words: Government intervention; Inefficiency; Taxation; Multinational

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1. Introduction

Government intervention is traditionally characterised as a non-strategic activity. A more realistic approach to model government is to treat it as an

* Corresponding author. Present address: Department of Economics, Cornell University, Ithaca, NY 14853, USA.

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entity that may have its own objective function but in strategic terms is no different from other agents, especially large ones like monopoly houses and multinationals. In analysing its interaction with large firms it may be best to treat the government as just another agent in a strategic environment and analyse the properties of Nash equilibria and some of its refinements. Indeed, instances where a multinational far exceeds its host country in economic power are numerous, especially in the Third World. Kindleberger (1984) and Casson (1987) are two examples from a vast literature that views multinational and government interaction in bilateral terms.

The central aim of the paper is (i) to characterise the Nash equilibrium of the government–firm game (ii) to demonstrate that 'strategic' inefficiency (explained in this section and formally defined later) is pervasive in these models whenever government uses ad valorem taxes, and (iii) if the government's choice between ad valorem tax and specific or per-unit tax is endogenised, then in a perfect equilibrium the government *will*, in fact, choose the ad valorem system. Observe that (iii) reinforces the significance of (ii).

In Section 2 the model is outlined and the existence of equilibrium is established. We consider an industrial situation in which there is a firm that sets price in order to maximise profits, and a government that sets ad valorem tax rates in order to maximise revenue collection. Much of the existing public economics literature analyses the consequences of changing tax rates (see, for example, Stern, 1987) and could be thought of as a step towards the Stackelberg-type analysis of government–firm interaction. We focus on the Nash equilibrium of such a game.

The revenue-maximisation assumption of the government deserves comment. Firstly, our paper shows how to model government–firm interactions and, therefore, could act as a guide to other models where the government's maximand is different. Secondly, and more importantly, we believe that governments have different objectives when they undertake different activities and when it comes to taxation policy a typical government's objective does turn out to be revenue-maximisation. This could be because, on the expenditure side, there are large precommitments such as defense, judiciary and administrative expenses. Hence, when it comes to taxation, the objective becomes a simple one of trying to keep the budgetary deficit low; that is, to maximise revenue collection. Recently, von Furstenberg et al. (1986) find that the sequence 'spend now – tax later' has much more empirical validity than the conventionally accepted pair of 'spend and tax jointly'. Finally, our assumption fits well standard models of bureaucratic behaviour (see, for example, Niskanen, 1971; McGuire et al., 1979).

In Section 3 a geometric technique of analysis is developed and our central result – that of strategic inefficiency – is established geometrically in the linear special case. While results of inefficiency are well known in the public economics literature, what we establish is different. In a *subgame perfect equilibrium* of a two-period model we show that a firm may commit itself to a 'high' cost function (by, for example, signing high-wage contracts with workers, buying outdated technology, or having a cumbersome management structure), which is very distinct from choosing a suboptimal point on the given cost curve. We refer to this as 'strategic inefficiency'.[1]

The strategic-inefficiency result is extremely robust. Section 4 generalises the result by dropping not only linearity but also differentiability, and proves a formal mathematical theorem.

In Sections 5 and 6 we examine the robustness of these results to alternative specifications. In Section 3 we assume that in period 1 the firm chooses its technology (cost function), and in period 2 the firm and government choose price and tax rate simultaneously. In Section 5 we consider a three-period model in which the sequence of moves is: cost function first, tax rates second, and price third.

In Section 6, the government is allowed (i) to choose between specific or per-unit tax rates and ad valorem taxes and (ii) to have a social-welfare-type objective function. It is interesting to note that the results of the paper remain valid even under these modifications.

Our model should be treated as illustrative but we believe that it provides a rich base for further theorising about the consequences of strategic government intervention. We feel that, unlike in many advanced industrialised countries, the main strategic interaction of a firm, especially a multinational giant in a developing economy, is not vis-à-vis other firms but vis-à-vis the government.

2. Definitions and the basic framework

Let $q(\cdot)$ be the *demand function*. Thus if w is the price faced by consumers, $q(w)$ denotes the aggregate demand for the good. We denote $q(0)$ by Q. We assume that (i) the demand function is continuous and $q(\cdot)$ is zero if the price exceeds a given number P and (ii) the demand curve is downward sloping.

Denoting the *inverse demand function* by $r(\cdot)$ we shall assume that (iii) the total revenue function $x \cdot r(x)$ is concave in x.

[1] This inefficiency is different from the technical and allocative inefficiencies studied in the literature, e.g. Farrell (1957). The cost-raising feature is common to both types of inefficiency. Strategic inefficiency as discussed in this section is distinct from slack or X-inefficiency (see, for example, Selten, 1986; Leibenstein, 1987).

The cost function, $c(\cdot)$ is (iv) continuous, convex and has zero fixed cost and (v) there exists a positive real number, b, such that $c(x) \geq bx$, for all x.[2]

In our model, there is a firm (a monopoly) which chooses the *producer price*, p, to maximise profit, π, and the government chooses an ad valorem tax rate, t, to maximise tax revenue, R. (We later consider modifications of this.) Given p and t, the *consumer price* is given by $(1+t)p$ and the aggregate demand for the good is given by $q((1+t)p)$.

The *firm's profit*, π, is given by

$$\pi(p,t) = pq((1+t)p) - c(q((1+t)p))\,. \tag{1}$$

The *government's revenue*, R, is given by

$$R(p,t) = tpq((1+t)p)\,. \tag{2}$$

We define (p^*, q^*) to be an *equilibrium* if

$$\pi(p^*, t^*) \geq \pi(p, t^*)\,, \quad \text{for all } p$$

and

$$R(p^*, t^*) \geq R(p^*, t)\,, \quad \text{all all } t\,. \tag{3}$$

Theorem 1. In the above model, given assumptions (i)–(v) *an equilibrium exists.*

A proof of this appears in the appendix.

3. A geometric characterisation and strategic inefficiency

Let us show a strikingly simple characterisation of the above equilibrium in the linear special case. Assume that the demand function is linear and the marginal cost curve is horizontal. These are illustrated in Fig. 1.

Let PA denote the marginal revenue curve. Through point Q, draw a ray in the north-west direction. Let B be the point where the ray cuts the vertical axis, D the point where it cuts the marginal cost curve, and E where it cuts the marginal revenue. Now, consider the ray that has the property, $BE = ED$. Let BQ be such a ray. From E draw a horizontal (resp. vertical) line and mark its intersection with the axis as p^* (resp. q^*), and let $t^* = OP/OB - 1$ and p^*_m be the corresponding market price.

[2] In the light of the convexity assumption, this implies that marginal cost is bounded away from zero.

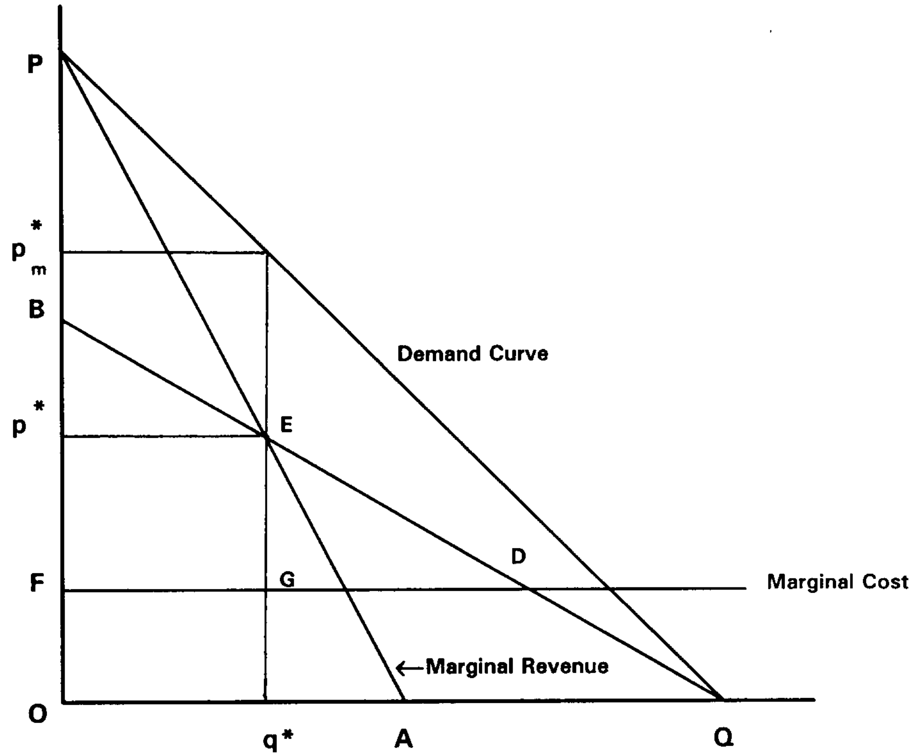

Fig. 1.

Assertion. (p^*, t^*) *is a Nash equilibrium in the one-period model of Section* 2.

We shall refer to this method of spotting the Nash equilibrium in the linear case as the *bisection rule* (since it involves bisecting the line BD) and the ray BQ, which has the property that $BE = ED$, will be called the *equilibrium ray*.

Proof of Assertion. First check how the firm chooses p, given t. In Fig. 2, let PQ be the demand curve and t the ad valorem tax rate. Let TQ be a line such that its height is

$$\left(\frac{1}{1+t}\right)$$

of the height of the demand curve. Hence TQ shows for each q the producer

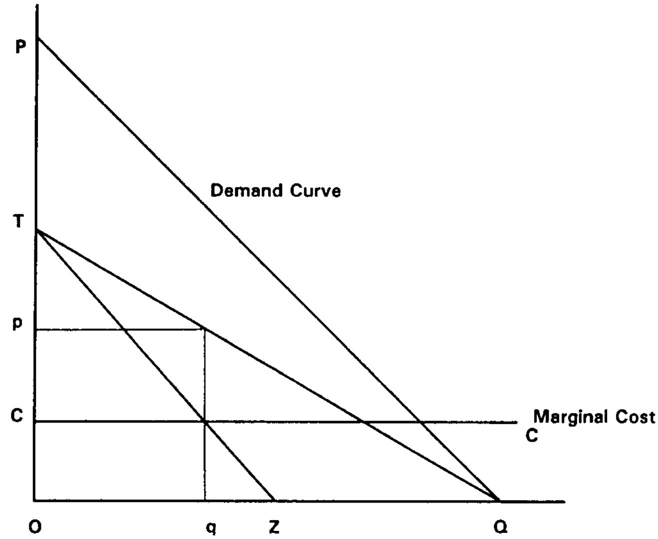

Fig. 2.

price. Treating this as a demand curve, the equilibrium is determined by the intersection of the 'marginal revenue' curve (line TZ in Fig. 2), with the marginal cost curve (line CC). In what follows, we shall refer to a curve like TQ as the *t-demand curve*. It will shift as the tax rate, t, changes.

Now return to Fig. 1. Let the tax rate be such that the t-demand curve is BQ. Since $BE = ED$, hence, $FG = GD$. Hence, the marginal revenue for BQ will intersect the marginal cost curve at point G. Thus, the firm's optimal choice of producer price is p^*.

Next, we examine the government's choice of t given producer price p. In Fig. 3, the producer price is p. If the government sets the tax rate at t, its revenue collection will clearly be equal to $PDBC$. For a given producer price p, the tax revenue is given by a rectangle sitting on the line pC and with a corner on the demand curve PQ. Clearly, this is maximised where the marginal revenue curve, PA, intersects pC, at the tax rate $\hat{t}$ where $\hat{t}$ is implicitly defined by $(1 + \hat{t})p = OF$. It is immediate that if the producer price is set at p^* in Fig. 1, the government's optimal choice of tax rate t^* is equal to $(p_m^*/p^*) - 1 = (OP/OB) - 1$. This completes the proof.

It is easy to see that $BE = ED$ is also a *necessary* condition for equilibrium. It is obvious from this and from an inspection of Fig. 1 that the equilibrium is unique in the linear case.

Using this diagrammatic characterisation, some properties of the linear case are immediate.

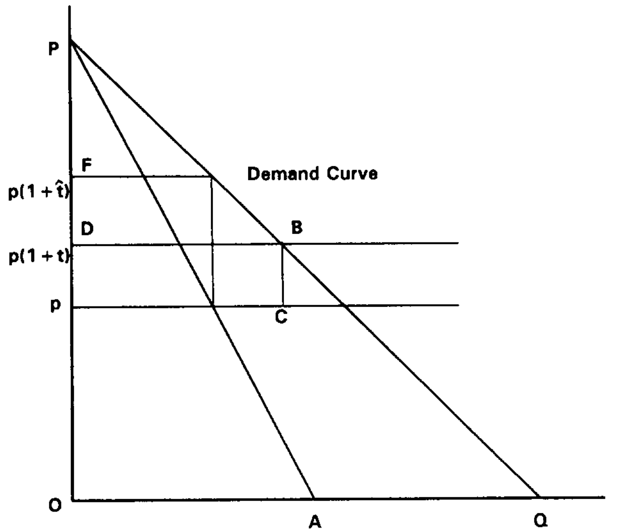

Fig. 3.

(1) If a firm has a lower marginal cost of production, it will face a higher ad valorem tax rate, charge a lower price and produce a larger amount.

(2) Equilibrium output is less than what would have been produced by a traditional monopolist, and the consumer's price higher.

We can now proceed to give a first view of the strategic-inefficiency result. It will be shown that when a firm plays strategic games with the government, as in the above model, it may be in the firm's interest to be inefficient. Unlike in some existing models (e.g. Seade, 1987) here the firm's choice is guided by strategic considerations vis-à-vis the government.

To establish this result we need to assume that there are two periods. In the first period the firm chooses a technology; that is, a cost function, from an exogenously given set of available cost functions. In the linear case, we could equivalently assume that the firm chooses one from an available set, $\bar{C}$, of cost functions. Thus, $\bar{C}$ is a set of positive numbers, each depicting a fixed marginal cost that the firm can choose to have by choosing a certain available technology.

The firm chooses $c \in \bar{C}$ in period 1 and then the firm and the government play the game described above in period 2. $c^* \in \bar{C}$ is an 'equilibrium' choice of firm 1 if the resulting equilibrium in period 2 gives the firm at least as large a profit as it can get by choosing any other $c \in \bar{C}$.[3]

If $c, c' \in \bar{C}$ such that $c > c'$, then (assuming zero fixed cost) c is clearly a

[3] Similar ideas have been used in models of entry-deterrence (Spence, 1977; Dixit, 1980; Basu and Singh, 1990).

more inefficient technology. We will say that the model exhibits strategic inefficiency if $\bar{C} = \{c, c'\}$ such that $c > c'$, but the firm chooses c. The equilibrium idea being used here is that of subgame perfection; this is discussed formally in the next section.

Using the geometric technique developed above, it can be nicely demonstrated that, at least in the linear case, strategic inefficiency is indeed a possibility. In Fig. 1, the marginal cost curve is FD and the firm's profit in equilibrium is given by the area p^*EGF. Turning to Fig. 4, suppose there is a new marginal cost curve, $F'D'$ which is lower than FD. Since $BE = ED$, it follows that $BE < E\hat{D}$. Hence, by the bisection rule, the new equilibrium will be on a flatter ray, QB', such that $B'E' = E'D'$.

Now consider a sequence of increasingly efficient technologies that lower the marginal cost curve. In particular, consider a sequence of constant marginal cost curves converging to the zero marginal cost curve.

Since $OA = AQ$, as marginal cost goes towards zero, the equilibrium ray clearly goes towards OQ. Further note that for marginal cost FD, the firm's profit is less than the area OBQ. But as marginal cost goes to zero, BQ tends to OQ and hence the firm's profit goes towards zero.

Therefore, we can pick a marginal cost curve from the sequence of curves going towards zero, which generates a smaller profit than p^*EGF in Fig. 1. Hence if the only two technologies available to the firm are the one generating the marginal cost curve we just picked, and the one that generated FD in Fig. 1, the firm will pick the latter even though it is more inefficient.

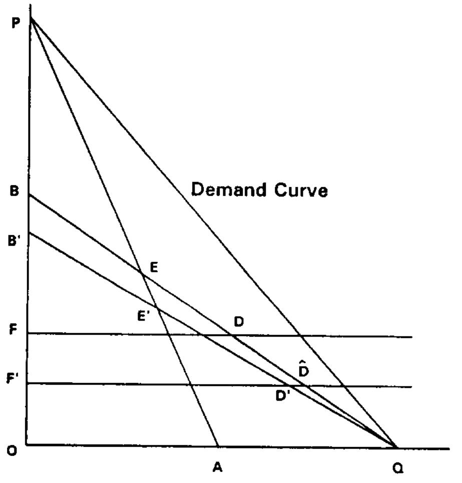

Fig. 4.

4. Strategic inefficiency: A general theorem

The aim of this section is to generalise the strategic-inefficiency claim of the previous section. As before, in period 1, the firm chooses a technology, embodied here entirely in a cost function, from an available set ζ, of cost functions. In period 2, with the cost function given, the firm and government interact as in the model of Section 2. The equilibrium concept that we use is that of (subgame) perfection.

For a formal definition we proceed as follows: Let ϕ be a *correspondence* such that for all $c \in \zeta$, $\phi(c)$ is the set of all (p, t) which is an *equilibrium* (in the sense of Section 2) when the cost function is c.

Then (c^*, p^*, t^*) is a *perfect equilibrium* if and only if there exists a *selection* ψ in ϕ such that

$$(p^*, t^*) = \psi(c^*) ,$$

and

$$p^*q((1+t^*)p^*) - c^*(q((1+t^*)p^*)) \geq pq((1+t)p) - c(q((1+t)p)) ,$$
$$\text{for all } (p, t, c) \in \{p', t', c') \mid (p', t') = \psi(c'), c' \in \zeta) .$$

Given two cost functions c and $\hat{c}$, we shall say that the former is *less efficient* than the latter if, for all $x \in [0, \infty)$, $c(x) > \hat{c}(x)$. We ask, can a cost function c^* ever be part of a perfect equilibrium if c^* happens to be less efficient than another cost function, c, which is available? The next theorem makes it clear that the answer to this is in the affirmative.[4]

We say that a model exhibits *strategic inefficiency* if there exists a collection of cost functions, ζ, such that in a perfect equilibrium the cost, c, chosen by the firm, happens to be everywhere less efficient than some other available cost function in ζ. It is important to note that strategic inefficiency as defined is a property of the model. We demonstrate the existence of such inefficiency by restricting attention to the class of linear cost functions. Hence, the set, ζ, of admissible cost functions is a subset of $\{c(\cdot) \mid \forall x, c(x) = c \cdot x, \text{ where } c \in (0, \infty)\}$. We shall refer to these cost functions by their (constant) marginal cost.

Theorem 2. Models of government–firm interaction satisfying assumptions (i)–(v) exhibit strategic inefficiency.

[4] Such inefficiency can be explained in two ways: either as the direct outcome of profit-maximisation, or we could think of different firms as following different customary rules of behaviour and we use a natural selection argument, which is becoming popular in the literature (see Jacquemin, 1987), to explain why only the inefficient ones would survive in an environment described by our model.

Proof. This is broken up into three steps.

Step 1. Let us first define surrogate profit and revenue functions

$$\tilde{\pi}(q, t, c) = \left[\frac{r(q)}{1+t} - c\right] q , \tag{4}$$

and

$$\tilde{R}(p, q) = R\left(p, \frac{r(q)}{p} - 1\right) = \left[\frac{r(q)}{p} - 1\right] pq . \tag{5}$$

It is easy to check that $(\hat{p}, \hat{t})$ is a Nash equilibrium associated with $\hat{c}$, if and only if there exists a $\hat{q}$ such that

$$r(\hat{q})/(1+\hat{t}) = \hat{p} ,$$

and

$$\hat{q} \in \operatorname*{argmax}_{q \in [0, Q]} \hat{R}(\hat{p}, q) \cap \operatorname*{argmax}_{q \in [0, Q]} \tilde{\pi}(q, \hat{t}, \hat{c}) . \tag{6}$$

This alternative characterisation of the Nash equilibrium in the second period is now used to complete the proof.

Step 2. We now prove that for all $p \in (0, P)$, there exists c and t such that $0 < c < p$, and $(p, t) \in \phi(c)$. Let $0 < \hat{p} < P$ and choose any

$$\hat{q} \in \operatorname*{argmax}_{q} \tilde{R}(\hat{p}, q) . \tag{7}$$

Define

$$\hat{t} \equiv (r(\hat{q})/\hat{p}) - 1 ,$$

and

$$\hat{c} \equiv \hat{p}^2/r(\hat{q}) .$$

Then it will be shown that $(\hat{p}, \hat{t}) \in \phi(\hat{c})$. In the light of (7) and the alternative Nash equilibrium characterisation in step 1, all we need to show is that

$$\hat{q} \in \operatorname{argmax} \tilde{\pi}(q, \hat{t}, \hat{c}) . \tag{8}$$

Note that (8) is equivalent to

$$\left[\frac{r(\hat{q})}{1+t} - \hat{c}\right]\hat{q} \geq \left[\frac{r(q)}{1+t} - \hat{c}\right] q , \quad \text{for all } q ,$$

or

$$r(\hat{q})\hat{q} - r(q)q + \hat{p}(q - \hat{q}) \geq 0 . \tag{9}$$

If we go through the same process of writing out (7) in long-hand we arrive at (9). Hence (7) implies (8).

Step 3. Step 2 implies we can construct a sequence $\{(c_n, p_n, t_n)\}$ such that, for all $n, c_n > 0$, $(p_n, t_n) \in \phi(c_n)$ and $p_n > 0$; and $\lim c_n = 0$ and $\lim p_n = 0$. Clearly then, the firm's profit goes to zero as c_n goes to zero. It is easy to check that for some $c > 0$, the firm earns a positive profit in Nash equilibrium. Hence, we could construct a ζ such that the firm chooses an inefficient cost function from ζ in the perfect equilibrium. Q.E.D.

Strategic inefficiency may not occur if the demand function does not satisfy the assumptions in Section 2. This is true, for instance, with all constant-elasticity demand curves.[5] Since in such a case $q(0)$ is not defined, such cases lie outside our framework. Essentially, what seems to be the (rather weak) requirement for strategic inefficiency to occur is to have demand curves such that (a) the marginal revenue (if defined) 'is zero at some finite output', and (b) there exists a high enough P for which $q(P)$ happens to be zero.

5. Stackelberg equilibria

In this section, we examine alternatives to the Nash characterisation of the second-stage game. The two obvious cases are the two Stackelberg games: S^1, where the government first chooses t, followed by the firm's choice of p; and S^2, the opposite sequence. First, consider the linear example of Section 3. Using the arguments of Section 3 we can describe the firm's behaviour by a surrogate reaction function $q_F(t, c)$ which specifies the firm's choice of quantity for every (t, c). The firm actually chooses price but, given t, quantity and price have a one-to-one relation. Similarly $q_G(p)$ is the government's surrogate reaction function.

From our earlier discussion, we know that $q_G(p)$ coincides with the marginal revenue curve. The $q_F(t, c)$ curve can be derived geometrically. With c constant, for every post-tax demand curve, we determine the profit-maximising price–quantity pair in the usual manner. The locus of these points for different values of t is illustrated in Fig. 5 and slightly incorrectly labelled as the $q_F(t, c)$ curve. It is increasing and convex. It

[5] While this may be worked out algebraically there is a simpler explanation due to the fact that with ad valorem taxes the post-tax demand continues to be of the same elasticity. Hence, since the monopoly price–cost margin is determined by the elasticity of demand, the price is independent of the tax rate.

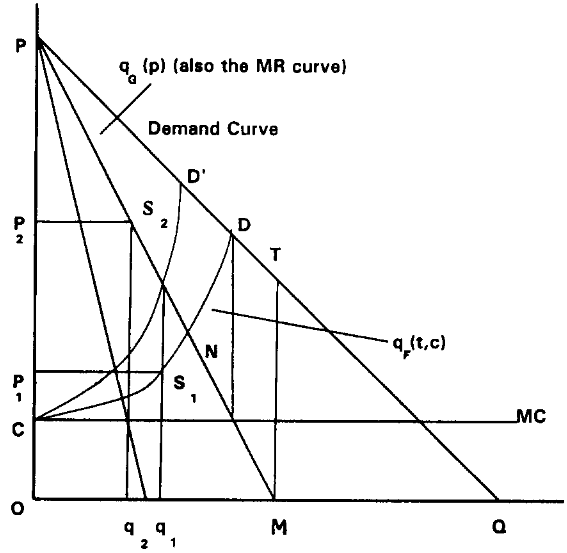

Fig. 5.

begins where the marginal cost touches the vertical axis and ends at the monopoly equilibrium on the demand curve.[6]

Consider first S^2. For each choice of p by the firm, the government will maximise revenue by choosing the corresponding point on the marginal revenue curve PM. The maximum profit for the firm can be located at S_2 by treating PM as the demand curve in a conventional monopoly. Price will be p_2 and tax will be given by the post-tax demand curve that goes through S_2.

S^1 is more complicated. The government knows that, given the firm's reaction function, it will invariably end up on curve CD. For each point on CD the government's revenue is equal to the area given by the rectangle formed by the demand curve and the vertical axis. So the government's problem is analogous to that of an ordinary monopoly facing the demand curve PQ and average cost curve CD. The solution uses the marginal curve (shown as CD') to this pseudo 'average cost' curve, CD. The intersection of CD' with marginal revenue PM identifies the equilibrium. The equilibrium tax rate is given by the t-demand curve going through point S_1 with price as P_1. The Nash equilibrium is the (point N) intersection of curves CD and PM in Fig. 5. It follows that with the government as Stackelberg leader the

[6] In the general, non-linear, case the shape of this could be quite irregular, violating monotonicity and continuity. We could have relaxed the assumption of linearity to concavity and $q_F(t, t)$ would be quite similar.

equilibrium tax rate will be greater than in the Nash equilibrium; price will be less and quantity produced would be less as well.

Supposing, now, that before playing each of these Stackelberg games the firm can commit itself to a cost function. It is easy to check that the inefficiency result does not hold in case S^2. But as we turn our attention to S^1, the strategic-inefficiency result comes back, as seen in Theorem 3 below. This is interesting given that S^1 is close to the standard characterisation of government–firm interactions.

Theorem 3. The perfect equilibria of three-period models of government–firm interaction, satisfying assumptions (i) to (v), in which choices are made in the sequence, (c, t, p), exhibit strategic inefficiency.

Proof. Let $R^{\mathrm{N}}(c)$ be the maximum revenue that the government can earn in a Nash equilibrium when the (constant) marginal cost of production is c. That is, $R^{\mathrm{N}}(c)$ solves

$$\operatorname{Max} R(p, t)$$

subject to

$$(p, t) \in \phi(c) .$$

It follows that if p is the price that prevails in a Nash equilibrium under c, then

$$R^{\mathrm{N}}(c) \geqslant \max_q \tilde{R}(p, q) = \max_q [r(q)q - pq] . \tag{10}$$

Next, note that if $\{p_n\}$ is a sequence converging to zero, then

$$\max_q [r(q)q - p_n q] = \max_q r(q)q . \tag{11}$$

We know from the proof of Theorem 2 that there exists a sequence $\{(c_n, p_n, t_n)\}$ such that $(p_n, t_n) \in \phi(c_n), c_n \to 0$ and $p_n \to 0$. It follows from (10) and (11) that

$$\lim_{n\to\infty} R^{\mathrm{N}}(c_n) \geqslant \max_q r(q)q .$$

But R^{N} can never exceed $\max r(q)q$ since that is the total profit in the system. Hence $\lim R^{\mathrm{N}}(c_n) = \max_q r(q)q$. Now, given marginal cost c, let $R^{\mathrm{S}}(c)$ be the revenue earned by the government if the government plays Stackelberg leader. Clearly, for all $c, R^{\mathrm{S}}(c) \geqslant R^{\mathrm{N}}(c)$. Hence $R^{\mathrm{S}}(c) = \max_q r(q)q$.

Let the firm's profit in this same Stackelberg be $\pi^{\mathrm{S}}(c)$. Since the total of profits and taxes cannot exceed $\max r(q)q$ we have $\lim \pi^{\mathrm{S}}(c_n) = 0$. Since there exist $c: \pi^S(c) > 0$ it follows that there will exist two cost functions c' and c'' such that the firm chooses the less efficient one. Q.E.D.

6. Extensions

In this section we examine the robustness of the results to relaxing some of the key assumptions of the models. Namely: (a) Would the inefficiency results be valid if the government's objective is not merely to maximize revenue but something more complicated like a weighted average of revenue and aggregate welfare? (b) How would the results be affected if instead of an ad valorem tax the government had to choose a specific or unit tax? and (c) Is there any reason to believe that in the context described in the paper the government would in fact use an ad valorem tax?

It will be seen that the results are indeed generalisable to such modifications. For (a) we have investigated several alternative formulations and the answer is 'yes'. We do not present the algebra here (it being available on request).

Turning to (b) and (c), unit taxes are easy to analyse and it can clearly be seen that inefficiency cannot arise in this case. However, the answer to (b) turns out to be less consequential in the light of the answer we get to (c). If we model government's choice of the type of tax, ad valorem versus per unit, as part of the "game" then we find that in equilibrium the government will choose to use an ad valorem tax. So, though the unit tax averts the strategic-inefficiency problem, in the government–firm game modelled here the government will use an ad valorem tax.

To begin with (c), we use the assumptions of Section 2 and, in addition, assume that the demand function is continuously differentiable (which considerably shortens the proofs). Consider a model where the government chooses between using A, an ad valorem tax, and U, a unit tax. We denote this choice by a variable, d, that takes values A or U, then it chooses the tax rate, t_d, and this is then followed by the firm's choice of price, p.

Given a choice, (d, t_d, p) the firm's profit is given by

$$\pi(A, t_A, p) \equiv \pi(p, t_A) \equiv pq((1+t_A)p) - c(q((1+t_A)p)) ,$$

and

$$\pi(U, t_U, p) \equiv \bar{\pi}(p, t_U) \equiv pq(p + t_U) - c(q(p + t_U)) ,$$

depending on whether $d = A$ or U.

The government's revenue is given by

$$R(A, t_A, p) \equiv R(p, t_A) \equiv t_A pq((1+t_A)p) ,$$

and

$$R(U, t_U, p) \equiv \bar{R}(p, t_U) \equiv t_u q(p + t_U) .$$

Note that a bar denotes the case of a specific or unit tax.

Theorem 4. In perfect equilibrium the government uses an ad valorem tax, i.e. $d = A$.

Proof. Define

$$p(t_A) = \underset{p}{\operatorname{argmax}}\ \pi(p, t_A) , \tag{12}$$

$$\bar{p}(t_U) = \underset{p}{\operatorname{argmax}}\ \bar{\pi}(p, t_U) . \tag{13}$$

Step 1. If t_A^* and t_U^* are such that

$$p(t_A^*)(1 + t_A^*) = \bar{p}(t_U^*) + t_U^* , \tag{14}$$

then $\bar{p}(t_U^*) > p(t_A^*)$. From (12) and (13) we know that

$$\frac{\partial \pi}{\partial p}(p(t_A^*), t_A^*) = 0 \quad \text{and} \quad \frac{\partial \bar{\pi}}{\partial p}(\bar{p}(t_U^*), t_U^*) = 0 .$$

Writing these out in full we get (omitting the asterisks):

$$q((1 + t_A)p(t_A)) + p(t_A)q'((1 + t_A)p(t_A))(1 + t_A) - c'(\cdot)q'(\cdot)(1 + t_A) = 0 ,$$

and

$$q(\bar{p}(t_U) + t_U) + \bar{p}(t_U)q'(\bar{p}(t_U) + t_U) - c'(\cdot)q'(\cdot) = 0 .$$

This, in turn, implies that

$$p(t_A) = \frac{c'(\cdot)q'(\cdot)(1 + t_A) - q(\cdot)}{q'(\cdot)(1 + t_A)} , \tag{15}$$

$$\bar{p}(t_U) = \frac{c'(\cdot)q'(\cdot) - q(\cdot)}{q'(\cdot)} . \tag{16}$$

Then (14) implies that

$$q((1 + t_A)p(t_A)) = q(p(t_U) + t_U) ,$$

and

$$q'((1 + t_A)p(t_A)) = q'(p(t_U) + t_U) .$$

For all t_A and t_U satisfying (14) $q'(\cdot)$ and $q(\cdot)$ in (15) and (16) are the same; therefore $\bar{p}(t_U) > p(t_A)$. This establishes the claim in Step 1.

Step 2. Next, if t_A and t_U satisfy (14), then

$$\bar{R}(\bar{p}(t_U), t_U) < R(p(t_A), t_A) .$$

This follows as an immediate consequence of $\bar{p}(t_U) > p(t_A)$, which was proved in step 1.

Step 3. It will now be proved that

$$\max_{t_A} R(p(t_A), t_A) > \max_{t_U} \bar{R}(\bar{p}(t_U), t_U) . \tag{17}$$

Let

$$\hat{t}_A \equiv \operatorname*{argmax}_{t_A} R(p(t_A), t_A) ,$$

and

$$\tilde{t}_U = \operatorname*{argmax}_{t_U} \bar{R}(\bar{p}(t_U), t_U) .$$

Let t_A be such that

$$p(t_A^*)(1 - t_A^*) = \bar{p}(\tilde{t}_U) + \tilde{t}_U .$$

Hence

$$\begin{aligned} \bar{R}(\bar{p}(\tilde{t}_U), \tilde{t}_U) &< R(p(t_A^*), t^*), \text{ by step 2} \\ &\leq R(p(\hat{t}_A), \hat{t}_A), \text{ by the definition of } \hat{t}_A . \end{aligned}$$

This proves Step 3. It is immediately obvious that choosing an ad valorem tax and setting the rate equal to $\hat{t}$ (and earning $R(p(\hat{t}), \hat{t})$ is part of a perfect equilibrium. Q.E.D.

In a game-theoretic model of industry, the sequence of moves often turns out to be critical for the results. What is surprising is that the above result is robust to some reasonable alterations of the sequence. The above theorem pertains to the case where, first (1) costs are chosen; next (2) the form of tax (i.e. A or U) is chosen by the government; then (3) the government chooses the tax rate (t_A or t_U); and, finally (4) the firm chooses the price. What happens, it may be asked, if the firm chooses cost between (2) and (3)? In other words, the sequence of moves is (2), (1), (3) and (4). It is easy to see that for a collection of cost functions C the theorem still goes through; that is, the ad valorem tax still emerges in equilibrium: Theorem 4 established this result for any cost function, say, c. Now take a collection of cost functions in the neighbourhood of c. Call the collection C. For a small enough neighbourhood C, even with sequence (2), (1), (3) and (4) the ad valorem system will be chosen in equilibrium because in the limiting case, where $C = \{c\}$, this sequence of moves is indistinguishable from the sequence (1), (2), (3) and (4), and the strict inequalities in the proof will continue to hold; and further in the latter case, we know from Theorem 4 that the ad valorem tax prevails.

Appendix

The purpose of this appendix is to prove Theorem 1. The proof is given by using a well-known variant of Nash's existence theorem for one-shot non-cooperative games.

In what follows, we use E to denote the set of real numbers and E_+ its non-negative subset. Lemma 1 states a property of composite functions. It is a special case of more general mathematical proposition, trimmed suitably for our purpose.

Lemma 1. Suppose a mapping $f: E_+ \rightarrow E_+$ had the property that for all $x, y \in E_+$, if $x \geqslant y$, then $f(x) \leqslant f(y)$. Let $f(E_+)$ be a subset of the interval $[a, b]$. If the mapping $g: [a, b] \rightarrow E$ is concave, then the composite mapping $g \cdot f$ must be quasi-concave.

Proof. Suppose the hypothesis of the lemma is true.

Let x and y be real numbers and $x < y$ and let $h \in [0, 1]$. Since f is negative monotonic

$$f(x) \geqslant f(hx + (1-h)y) \geqslant f(y) .$$

Hence, there exists $\alpha \in [0, 1]$ such that

$$f(hx + (1-h)y) = \alpha f(x) + (1-\alpha) f(y) . \tag{A1}$$

Since g is concave, g must be quasi-concave. Hence

$$g(\alpha f(x) + (1-\alpha) f(y)) > \min\{g(f(x)), g(f(y))\} . \tag{A2}$$

(A1) and (A2) imply

$$g \cdot f(hx + (1-h)y) \geqslant \min\{g \cdot f(x), g \cdot f(y)\} .$$

Lemma 2. The mapping $\pi(p, t)$ is quasi-concave in p and the mapping $R(p, t)$ is quasi-concave in t.

Proof. Define a new function $\tilde{\pi}$ as follows

$$\tilde{\pi}(x, t) \equiv \pi\left(\frac{r(x)}{1+t}, t\right) = \frac{xr(x)}{1+t} - c(x) .$$

Clearly, $\tilde{\pi}$ is concave in x, since $xr(x)$ and $-c(x)$ are both concave.

Given the definition of $\tilde{\pi}$, it follows that

$$\pi(p, t) = \tilde{\pi}(q((1+t)p), t) .$$

Hence, for a given t, π is a composite mapping of $\tilde{\pi}$ and q. Since q is inversely monotonic and $\tilde{\pi}$ is concave in the first variable, by Lemma 1 we know that $\pi(p, t)$ is quasi-concave in p.

The quasi-concavity of $R(p, t)$ in t is proved in the same manner by defining the following new function:

$$\tilde{R}(p, x) = R\left(p, \frac{r(x)}{p} - 1\right) = \left(\frac{r(x)}{p} - 1\right)pq . \quad \text{Q.E.D.}$$

Proof of Theorem 1. Let b be a positive real number such that for all $x \in [0, \infty)$, $c(x) \geq bx$. The existence of such a b is ensured by assumption (v) in Section 2. Define

$$T = \frac{P}{b} - 1 .$$

[Recall the definition of P from (i) in Section 2.]

Suppose now the firm and the government are playing a game of maximising respectively π and R. The firm chooses a p from the interval $[b, P]$ and the government chooses a t from the interval $[0, T]$.

Note that now

(A) each player chooses his strategy from a convex and compact set,

(B) the mappings π and R are continuous since q and c are continuous, and

(C) π is quasi-concave in p and R is quasi-concave in t, by Lemma 2.

By a well-known variant of Nash's theorem (see Friedman, 1977, p. 160, Theorem 7.4) it follows that this game must have a Nash equilibrium. That is, there exists $(p^*, t^*) \in [b, P] \times [0, T]$ such that

$$\pi(p^*, t^*) \geq \pi(p, t^*) , \quad \text{for all } p \in [b, P] , \tag{A3}$$

and

$$R(p^*, t^*) \geq R(p^*, t) , \quad \text{for all } t \in [0, T] . \tag{A4}$$

The only thing that remains to be proved is that the domain restrictions (price to $[b, P]$, and tax rate to $[0, T]$) are inessential: suppose (p^*, t^*) satisfy (A3) and (A4). Consider a $p \in [0, \infty) \backslash [b, P]$. If $p > P$, then $q((1 + t^*)p) = 0$. Hence, $\pi(p, t^*) = 0$. Suppose, now, $p < b$. Note that

$$\pi(p, t^*) < pq((1 + t^*)p) - bq((1 + t^*)p) < 0$$

by the definition of b, since $p < b$. Since $\pi(p, t^*) = 0$, we know by (A3) that

$$\pi(p^*, t) \geq 0 .$$

Hence,

$$\pi(p^*, t^*) \geq \pi(p, t^*)\,, \quad \text{for all } p \in [0, \infty)\backslash[b, P]\,.$$

Therefore, together with (A3), we have

$$\pi(p^*, t^*) > \pi(p, t^*)\,, \quad \text{for all } p \in [0, \infty)\,.$$

Suppose $t > T$. Then

$$\begin{aligned}(1+t)p^* &> (1+T)b, \text{since } p^* \geq b\,,\\ &= P, \text{by the definition of } T\,.\end{aligned}$$

This implies $q((1+t)p^*) = 0$.

Hence, $R(p^*, t) = 0$, by (2). Since $R(p, t)$ is non-negative for all $(p, t) \in [0, \infty) \times [0, \infty)$, it follows, $R(p^*, t^*) \geq R(p, t)$, for all $t > T$.

This and (A4) imply

$$R(p^*, t^*) \geq R(p^*, t)\,, \quad \text{for all } t \in [0, \infty)\,. \quad \text{Q.E.D.}$$

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