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Collusion in finitely-repeated oligopolies

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Explaining collusion in infinitely-repeated oligopolies is problematic because of the backward induction argument. The usual response to this has been to consider infinitely-repeated games and treat these as approximations of long, finite repetitions or to introduce a modicum of irrationality in firm decision-making. The present paper explores a third avenue which introduces some extensive form structure to the Cournot game itself. Hence the supergame constructed in this paper is one in which the stage game is itself an extensive-form game. In particular, the stage game is a Cournot oligopoly in which firms can observe each others initial production and respond by producing more before the product is sold on the market. With this structure it is possible to devise a discriminating trigger strategy which generates collusive behaviour among fully rational firms with finite lives.

1. Introduction

The explanation of collusion in finitely-repeated oligopolies is problematic because the backward-induction argument predicts the Cournot equilibrium in each period. However, it is intuitively troublesome to think of rational firms, year after year, making a lower profit because of this.[1]

Economists have responded to this conflict between intuitition and reason by constructing a whole range of models explaining collusion. One approach is to consider an infinitely-repeated oligopoly and *treat it* as an approximation for very long, finite repetitions [see Friedman (1971); Green and Porter (1984); Abreu, Pearce and Stacchetti (1986); Rotemberg and Saloner (1986)]. The second avenue is to retain the *finite*-repetition assumption as realistic

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[1]For a related problem concerning entry-deterrence Selten (1978) expresses a similar sentiment. So strong does he find the intuitive-argument's challenge to backward-induction that he describes this as a paradox and asserts that 'up to now I met nobody who said that he would behave according to the induction theory'.

but modify the assumption of individual rationality [see Radner (1980), and in a more abstract context, Kreps, Milgrom, Roberts and Wilson (1982)].[2]

There is a third line of analysis which seems to have been ignored in the large literature on collusion. This involves giving more structure to the Cournot *stage* game. Once we allow for this, collusive behaviour becomes explicable without having to sacrifice the assumption of *finite* length of interaction and full rationality (in fact, its common knowledge) among the firms. The aim of this paper is to explore this line of explanation.

Production takes time and it seems reasonable to suppose that an oligopolist can observe whether its competitors are planning to produce a lot or a little; and can respond to this by adjusting its own production plans before the product is finally made available on the market. I shall in the present paper allow for this kind of structure in a somewhat stylized manner, which is a variant of Saloner (1987). This entails thinking of each stage game as broken up into two substages. In the first substage each firm produces some amount. They observe this and in the second substage produce more or dispose of any amount of the output produced in the first substage. Then their total production is offered on the market and price and profits are determined in the usual Cournot style. I shall refer to this as the *modified* Cournot game. Note that the modified Cournot game is itself an extensive-form game. The model in section 2 which combines Saloner (1987) and Friedman (1985) is an illustration of this.

If the modified Cournot game is played a finite number of times, collusion becomes possible under subgame perfection. We have to, however, be careful in defining 'subgame perfection' since the full game consists of repetitions of an *extensive*-form game. Hence, we have to consider histories of not just n stage games but also n stage games and a 'half' of the $(n+1)$th stage game.

My model can be modified in many ways: by using different trigger strategies, such as in Benoit and Krishna (1985), and other stage games, such as in Basu (1990). Recently robustness questions have been raised about Saloner's model [see Pal (1991)]. Section 3 argues that those features of Saloner's model which are used here are relatively robust. The most untenable feature of Saloner's model is the assumption that it is impossible to dispose of goods once produced. Section 4 relaxes this assumption. Though the case I discuss is the polar one where destroying goods is costless, the *method* can be used to analyse more realistic intermediate cases.

Though the focus of this paper is on *collusion* in oligopolistic industries a subject of considerable interest in the industrial organization literature [see Green and Porter (1984); Abreu, Pearce and Stacchetti (1986)] one can raise more general questions concerning the *set* of outcomes that are supportable

[2]This raises some troublesome methodological questions [see, e.g., Binmore (1987); Basu (1990a)], but my concern in this paper is more mundane.

in equilibrium. Though subgame perfection in my model places more severe restrictions on strategies than in models where a *one* shot game is played repeatedly, a Benoit–Krishna type argument can be constructed even here. Hence, any payoff which is feasible and individually rational will, arguably, be supportable in equilibrium if the number of repetitions is large enough. This last qualification is extremely important and is the reason why Benoit and Krishna call their result a '*limit* perfect folk theorem'. While such a result is of game theoretic interest, its appeal in the industrial-organization framework considered here is limited since it hinges on our being able to repeat the stage game arbitrarily many times, which violates the 'spirit' of finiteness.

2. The model and collusive equilibria

2.1. Basic concepts and definitions

The aggregate inverse demand function facing the industry (in each period) is given by $p=p(x)$ and firm i's cost function is $c_i(x_i)=d_i x_i$, $i=1,2$. We shall assume throughout that the cost function is linear. If the total production of the firms are x_1 and x_2, then firm i's profit is given by

$$\pi_i(x_1,x_2)\equiv x_i p(x_1+x_2)-\mathrm{d}_i x_i,\ i=1,2. \tag{1}$$

The *reaction function* of i is defined as

$$\phi_i(x_j)\equiv \operatorname*{argmax}_{x_i}\pi_i(x_1,x_2),\ i\neq j,\ i=1,2. \tag{2}$$

It is being assumed that for all x_j, $\phi_i(x_j)$ is unique.

The output pair, $x^N=(x_1^N,x_2^N)$ will be called the *Cournot outcome*[3] if and only if $\pi_1(x^N)\geqq\pi_1(x_1,x_2^N)$, for all x_1 and $\pi_2(x^N)\geqq\pi_2(x_1^N,x_2)$, for all x_2. We shall throughout use x^N to denote the Cournot outcome. We define the Cournot profit as follows:

$$\pi_i^N\equiv\pi_i(x^N).$$

The *Stackelberg outcome with 1 as leader* is $\bar{x}=(\bar{x}_1,\bar{x}_2)$ if $\bar{x}_1=\operatorname{argmax}_{x_1}\pi_1(x_1,\phi_2(x_1))$ and $\bar{x}_2=\phi_2(\bar{x}_1)$. The Stackelberg outcome with 2 as leader is defined symmetrically. The profit of i when j is Stackelberg leader is denoted by π_{ij} and this is defined as follows:

[3] I purposely use the word 'outcome', instead of 'equilibrium', because the Cournot outcome is not what the present model predicts will happen necessarily. It is merely a definition which will come in handy later.

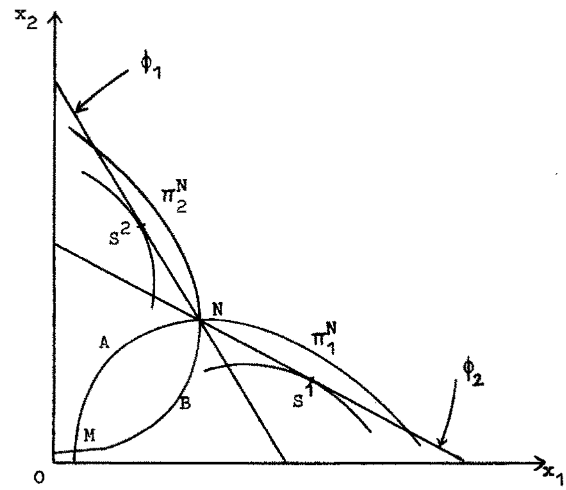

Fig. 1

$$\pi_{ij} \equiv \pi_i(\bar{x}),$$

where $\bar{x}$ is the Stackelberg outcome with j as leader.

Fig. 1 illustrates the above functions and outcomes, with N denoting the Cournot outcome and S^i the Stackelberg outcome with i as leader. The curves marked π_i^N, $i=1,2$, are iso-profit curves through N.

Throuhgout the analysis I shall assume that the demand function is sufficiently well-behaved for the following to be true.

(i) For each i, π_i is continuous and strictly quasi-concave with respect to x_i.
(ii) $\pi_1(x_1, \phi_2(x_1))$ is strictly quasi-concave with respect to x_1 and $\pi_2(\phi_1(x_2), x_2)$ is strictly quasi-concave with respect to x_2.
(iii) $\partial\phi_i/\partial x_j > -1$, $i \neq j$, $i=1,2$.

Condition (iii) implies that reaction functions are 'stable' in the textbook sense and the Nash outcome is unique. Condition (ii) has the advantage of ensuring that there is a unique Stackelberg equilibrium and that the Stackelberg leader's profit increases monotonically as we move along the follower's reaction curve towards the Stackelberg equilibrium. This condition is used in Basu and Singh (1990).

2.2. The stage game, G

As discussed above, we want to give some temporal structure to the usual Cournot game by allowing for the fact that production takes time and firms can spy on each other's production effort and make suitable adjustments.

This is captured formally by using Saloner's (1987) formulation. Asume that each period or stage is broken up into two substages. In substage 1, the firms produce (x_1^1, x_2^1). Then, having observed this, in substage 2, they produce (x_1^2, x_2^2). We assume, $x_i^t \geqq 0$, for all t, for all i. This implies that in the second substage, firms can only add to their output. This is a temporary assumption which allows me to use Saloner's theorem. It is possible to allow firms the freedom to destroy output in the second substage without losing my central claim concerning collusion. This is shown in section 4.

If s_i is a *strategy* of firm i, then s_i specifies a production level, x_i^1, in substage 1 and a function on all output pairs such that $s_i(x_1^1, x_2^1)$ is the output i would produce in substage 2 if in substage 1, (x_1^1, x_2^1) is produced. The set of all strategies of i is denoted by E_i. Given a strategy pair $s \in E_1 \times E_2$ the output that is produced by player i in substage t is denoted by $x_i^t(s)$. We define

$$\begin{aligned} &x^t(s) \equiv (x_1^t(s), x_2^t(s)), \text{ and} \\ &x_i(s) \equiv x_i^1(s) + x_i^2(s). \end{aligned} \tag{3}$$

Note that these definitions imply $s_i(x^1(s)) = x_i^2(s)$.

The *payoff function*, $\tilde{\pi}_i$, of firm i in the game, G, is a real-valued function on $E_1 \times E_2$ such that for all $s \in E_1 \times E_2$,

$$\tilde{\pi}_i(s) \equiv \pi_i(x_1(s), x_2(s)),$$

where π_i is given by (1), above.

The subgame perfect equilibrium of G is defined in the usual way. The stage game will be referred to as the *extensive Cournot game*.

2.3. The supergame, (G, T)

The supergame, (G, T), is simply the extensive-form game created by a T-fold repetition of the stage game, G. I shall describe (G, T) in a slightly unusual way which makes no explicit reference to the stage game. This is merely a matter of convenience.

Note that (G, T) involves $2T$ substages which we shall refer to as substages $1, 2, \dots, 2T$. Every consecutive pair of substages, $(t, t+1)$, where t is odd, constitutes a stage.

Let x_i^t be firm i's output in substage t. We write $x^t \equiv (x_1^t, x_2^t)$. A *t-substage history* is a sequence, $(x^1, \dots, x^t)$, of outputs in all substages up to the tth one. I shall at times refer to a t-substage history as $h(t)$. Let $H(t)$, $t > 1$, be the set of all t-substage histories. Define $H(0) = \{\phi\}$ and $H = \bigcup_{t=0}^{2T-1} H(t)$.

A strategy, f_i, of firm i in the supergame, (G, T), is a mapping, $f_i: H \to R_+$, where R_+ is the set of non-negative real numbers. $f_i(h(t))$ denotes the output i will produce in substage $t+1$, given that the history thus far is described by $h(t)$; and $f_i(\phi)$ is i's output in substage 1, that is, at the start of the supergame.

Let F_i be the set of all strategies of i in (G,T). If $f \in F_1 \times F_2$, then $\sigma(f)$ denotes the path of outputs that would be produced if f is the strategy pair being played; $\sigma(f)$ is defined in the usual way.

Finally, we have to define the *pay-off function*, P_i, of player i. Let D be the set of odd numbers in $\{1,2,\ldots,2T\}$. If $f \in F_1 \times F_2$, and $\sigma(f)=(x^1,\ldots,x^{2T})$ then

$$P_i(f) \equiv \sum_{t \in D} \pi_i(x_1^t + x_1^{t+1}, x_2^t + x_2^{t+1}).$$

The use of the stage game is implicit in the specification of P_i.

The *average pay-off* earned by i in (G,T) while playing f is denoted by $P_i^T(f)$ and defined as $P_i^T(f) \equiv P_i(f)/T$.

The equilibrium criterion that will be used is subgame perfection. To define this formally we need some more notation. Given any strategy-pair, f, and a history , $h(t)$, the path generated in the $2T-t$ remaining substages is defined in the usual way and denoted by $\sigma(f,h(t))$. I shall use $\bar{P}_i$ to define payoffs in subgames. If f is the strategy-pair being used and $h(t)$ is the history after which the $(2T-t)$-substage subgame is being considered, then $\bar{P}_i(f,h(t))$ is i's pay-off in the subgame. This is defined as follows.

$$\bar{P}_i(f,h(t)) \equiv \sum_{t \in D} \pi_i(x_1^t + x_1^{t+1}, x_2^t + x_2^{t+1}),$$

where $h(t)=(x^1,\ldots,x^t)$ and $\sigma(f,h(t))=(x^{t+1},\ldots,x^{2T})$.

A strategy-pair f^* is a *subgame perfect equilibrium* of (G,T), if and only if, for all $h(t) \in H$, for all $i \in \{1,2\}$ and for all $f_i \in F_i$,

$$\bar{P}_i(f^*,h(t)) \geqq \bar{P}_i(f^*/f_i, h(t)),$$

where f^*/f_i is the strategy-pair formed by replacing the ith element of f^* with f_i.

2.4. The theorem

Let $(x_1^*, x_2^*) = x^*$ be such that $\pi_i(x^*) \geqq \pi_i^N$, $i=1,2$. In other words, x^* is any point in the area enclosed by MANBM in fig. 1.

Theorem 1. (a) There exists T such that there is a subgame perfect *equilibrium, f^*, in (G,T) which results in x_i^* being produced by i in the first stage game. That is, the outputs produced by i in substages 1 and 2 add up to x_i^*.*

(b) For all $\varepsilon>0$, there exists T such that f^* is a subgame perfect equilibrium of (G,T) and $P_i^T(f^*)>\pi_i(x_1^*,x_2^*)-\varepsilon$.

Part (a) of the theorem asserts that in the early stage games collusive behaviour can be supported under subgame perfection. Part (b) asserts that as the supergame is made longer and longer, the stages with collusive behaviour can be so much more numerous than the stages with non-collusive outcomes that the average (i.e., per-stage) pay-off converges to the pay-off under collusion.[4]

Since the joint-profit maximizing locus in the usual Cournot game (i.e., the contract curve) passes through the area MANMB, the theorem suggests that joint-profit-maximizing outcomes are possible (in the limit) under full, individual rationality and finite repetitions of the modified Cournot game.

2.5. The proof

A critical input in the proof of Theorem 1 is a proposition of Saloner's (1987), which is restated below as Lemma 1. This is concerned with outcomes in the stage game, G, described above. Let $(x_1(i),x_2(i))$ be the Stackelberg outcome with i as leader, and, as before, let (x_1^N,x_2^N) be the Nash outcome.

Lemma 1. In the stage game, G, each of the following three production paths is the outcome of a strategy pair, which is subgame perfect:

(a) In the first substage, the firms produce $(x_1(1),x_2(1))$ and in the second substage they produce $(0,0)$.

(b) In the first substage, the firms produce $(x_1(2),x_2(2))$ and in the second substage they produce $(0,0)$.

(c) In the first substage, the firms produce $[x_1^N,x_2^N]$ and in the second substage they produce $(0,0)$.

This lemma asserts that the Cournot outcome as well as the two Stackelberg outcomes can be supported under subgame perfection if the Cournot game is played in two stages as in G. For a detailed proof the reader is referred to Saloner (1987). I shall give here a sketch of the argument which will help prepare the ground for a generalization of Saloner's result to the case with free disposal of output.

[4]This means that the average profit earned by a firm will be between the joint-profit maximising solution and the standard Cournot outcome, converging towards the former only as the time horizon becomes long. It is instructive to compare this with Slade's (1987) interesting empirical study based on competition among gasoline service stations in Vancouver. She finds that the stations earn less than the monopoly profit but more than in 'non-cooperative single-period solutions'.

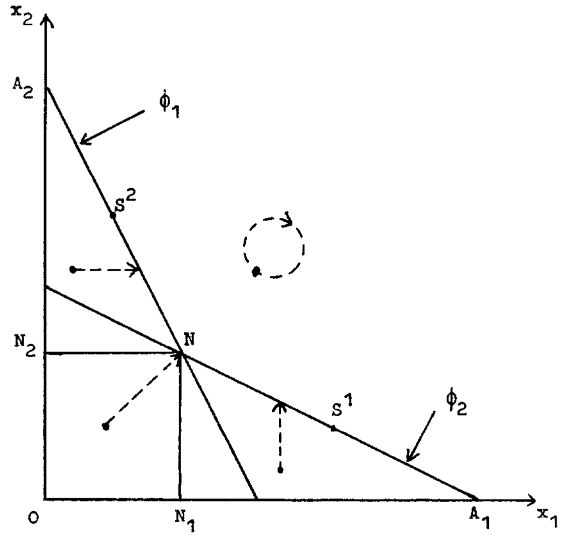

Fig. 2

To prove Lemma 1 suppose $(x_1^1, x_2^1) = x^1$ has been produced in substage 1. What will happen in substage 2 depends on which of the following four regions of fig. 2, x^1 happens to be in: Region ON_1NN_2; Region N_1NA_1; Region N_2NA_2; Region north-east of A_2NA_1. Under subgame perfection, if x^1 is in region ON_1NN_2, substage 2's output must be such that total output occurs at N; if x^1 is in region $N_1NA_1(N_2NA_2)$ substage 2's output must be such that total output occurs on firm 2's (firm 1's) reaction function vertically above (horizontally to the right of) x^1. If x^1 is in the north-east of A_2NA_1, then total output remains at the same place. The dashed lines with arrows summarise this in fig. 2.

The proof of this is simple, and requires keeping in mind that subgame perfection implies Nash equilibrium play in the second substage and if x^1 has been produced in substage 1, then only the region north-east of x^1 is feasible in substage 2.

Now turn to substage 1. Suppose firms 1 and 2 plan to produce S^1 in substage 1. If 1 deviates, i.e., the production in substage 1 is a horizontal displacement from S^1, then, given what we already know will happen in substage 2, the final outcome will occur either somewhere on the line NS^1 or somewhere to the right of S^1. In either case, firm 1 cannot do better through a deviation. It is easy to check that neither can 2 do better. Hence, S^1 is an outcome of a subgame perfect equilibrium. The proofs for S^2 and N are similar. This completes the proof of Lemma 1.[5]

[5]Actually all points on the line segments NS^1 and NS^2 can be supported under equilibrium.

The proof of Theorem 1, which will be stated only in outline, entails the construction of a suitable discriminating trigger strategy. Let x^* be as in Theorem 1. Consider the following 'path': In the first stage game x^* is produced – this is achieved by producing x^* in substage 1 and 0 in substage 2. Then on they produce x^N in each stage – the substage-wise break-up within each stage is achieved in a manner which is subgame perfect in G. The trigger strategy consists of proceeding on the above path as long as no deviation by a single player occurs at any substage. If i deviates alone in some substage, then in all remaining stages they play so as to achieve the Stackelberg outcome with $3-i$ as leader. As before, *within* each stage they employ strategies which are subgame perfect in G.

All this is standard and hence I desist from formalism. The part where care is needed is if i's deviation occurs at an odd numbered substage, t. This means a deviation occurs in the 'interior' of the stage game (something that cannot happen in the standard framework where the stage game is one-shot). The trigger strategy requires producing that amount in substage $t+1$ which would be a Nash equilibrium in the second substage of G, given that in the first substage the amount produced in t has been produced.

More formally suppose f^* is the trigger strategy being defined. Let x^t be the last element of a history $h(t)$, where t is an odd number. In case $h(t)$ involves only one deviation, and that by player j in period t, then f^* must be such that $f^*(h(t))$ is the Nash equilibrium output in the second substage of G in which x^t has been played in the first substage. As is evident from Lemma 1, this involves moves which are summarized by the broken lines and arrows in fig. 2.

Given the above trigger strategy a player can clearly not benefit from a deviation after stage 1 because from here on all play consists of repetitions of subgame perfect equilibria of G. By deviating in stage 1, a player gains in stage 1 but loses in all future stages. So if the future is long enough such a deviation can never be worthwhile. This completes the proof of part (a). Part (b) is now obvious and follows a standard argument.

3. Extensions and robustness

As explained earlier the model in section 2 is an *illustration* of a more general idea. The model can be extended and modified in several directions. A generalisation to the case of oligopoly with n firms is fairly easy and will not be pursued here.

One extension – which amounts to a robustness check on the Saloner model – is to allow for differences in the costs of production in the two substages of G. Return to the description of G and suppose the costs of producing one unit in substages 1 and 2 are respectively, c_1 and c_2. A recent paper by Pal (1991)[6] shows that Saloner's theorem depends critically on his assumption that $c_1 = c_2$. Consider first $c_1 < c_2$. It is easy to check that in this case the only subgame perfect equilibrium of G is the Cournot outcome. Hence in such a case my collusion argument would not be possible to construct.

What happens in the other case, i.e. $c_1 > c_2$? Pal shows that if the gap is not 'too large', then the only subgame perfect equilibria of G are the two Stackelberg outcomes. While this does damage the 'continuum claim' in Saloner (1987), *it retains the feature of Saloner's model which is needed for my model of section 2.* While it is true that the simple trigger strategy of Friedman (1985) cannot be used to sustain collusive behavior, we can use more-sophisticated but standard trigger strategies which require rotating between the two Stackelberg outcomes in the last games. And note that if c_1 is just a little higher than c_2, there will exist open neighborhoods around c_1 and c_2 such that as long as the costs are anywhere in these neighborhoods the two Stackelberg outcomes will be supportable by subgame perfect equilibrium strategies in the extensive Cournot games.

There is a more general robustness issue concerning Saloner's model. Let us return to the game G with $c_1 = c_2$. In this section I shall call a strategy-pair a *sturdy equilibrium* if it is subgame perfect and neither of these strategies is weakly dominated. Clearly, if a strategy-pair is not a sturdy equilibrium, it cannot be a *trembling-hand* perfect equilibrium. It is possible to show that only the two Stackelberg outcomes can be supported by *sturdy-equilibrium* strategies. In other words, all the other Saloner equilibria entail the use of a weakly dominated strategy by at least one player. Fortunately, as the remarks in the above paragraph indicate, this does not change the claims of the present paper concerning the possibility of collusion.

4. Free disposal

All this time it was being assumed that goods, once produced, cannot be destroyed. This is of course an unrealistic assumption. The aim of this section is to demonstrate that the claims of this paper remain intact even if we allow for free disposal. Consider the description of the game G in section 2.2. We will here relax the assumption that $x_i^2 \geqq 0$ for all i. Instead, it will be assumed that $x_i^2 \in [-x_i^1, \infty)$. That is, in the second substage a firm can destroy any amount of the production undertaken in the first substage or produce more. In addition, destruction of output is costless, that is, we have free disposal. I shall call this game G^*.

Before going into the analytics note that G^* is best viewed as a polar case of a more realistic model in which disposal is not completely costless. There

[6]For some related research currently being done at Princeton, see Banerji and Cooper (1991).

are products which, once produced, are difficult to destroy and costly to store. Hence, in reality, whereas production usually entails substantial costs, even destruction involves some costs, though generally less than what is incurred for production. Hence, the alternative assumptions of destruction being impossible and it being costless are both polar cases. We are about to analyse what seems to me to be the one closer to reality, namely, the latter case. More importantly, once my *method* of analysis is understood, it will be evident that the non-polar case can be analysed by the same method. The technique consists of using the real and the 'pseudo reaction function' (defined below), in a manner originally used by Dixit (1980) [see also Basu and Singh (1990)]. As will be clear later, the non-polar case is basically one where the relevant pseudo reaction function moves further away from the real one.

Turning now to the analysis of G^*, I shall briefly sketch a proof which demonstrates that even after we allow for free disposal, the stage game has at least three subgame perfect equilibria: (i) the usual Cournot outcome, (ii) an equilibrium where firm 1 earns a smaller profit than in the Cournot outcome and (iii) one where 2 earns a smaller profit than in the Cournot outcome. Once this is proved, it is obvious that a trigger strategy can be constructed to support collusive behaviour under subgame perfection.

To sketch a proof of the claim made in the above paragraph, let me introduce some new terminology. By a *pseudo-reaction function* of firm i, I shall here mean a reaction function of i under the assumption that i's cost of production is 0. If $\bar{\phi}_i(\cdot)$ is i's pseudo-reaction function then

$$\bar{\phi}_i(x_j) = \underset{x_i}{\operatorname{argmax}}\, x_i p(x_1 + x_2).$$

Contrast this to (2). In fig. 3, firm 1's reaction and pseudo-reaction functions are both illustrated.

Suppose firm 1 produces x_1^1 in substage 1. Then, given free disposal, firm 1's reaction function in terms of total outcome produced at the end of substage 2 is given by the line ABCD′. This is because to produce more than x_1^1, the marginal cost is c_1 and DD′ is the relevant reaction function. To produce a total amount less than x_1^1, the marginal cost is zero. In other words, if firm 2 produces a total output of x_2, firm 1's optimal response in terms of the total output produced in the two substages, given that it has already produced x_1^1 in substage 1, can be read off the line ABCD′. Firm 2's aggregate reaction function is similarly worked out.

Now it is easy to work out for each substage 1 production, $(x_1, x_2) \equiv x^1$, where the Nash equilibrium at the end of substage 2 will occur. Following the method used in section 2.5, the result of what happens in substage 2 after every possible history is summed up in fig. 4. Essentially there are nine

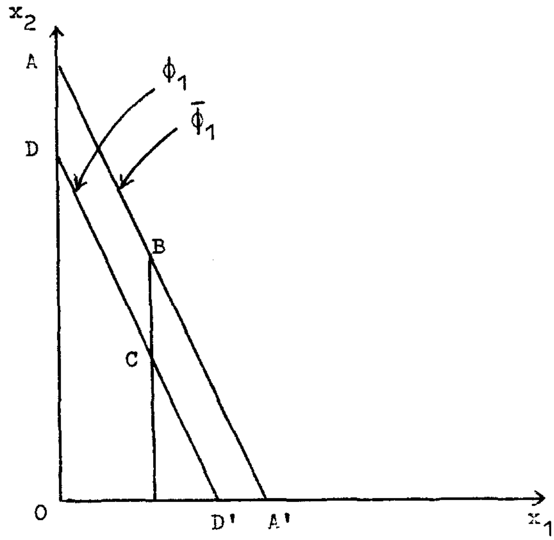

Fig. 3

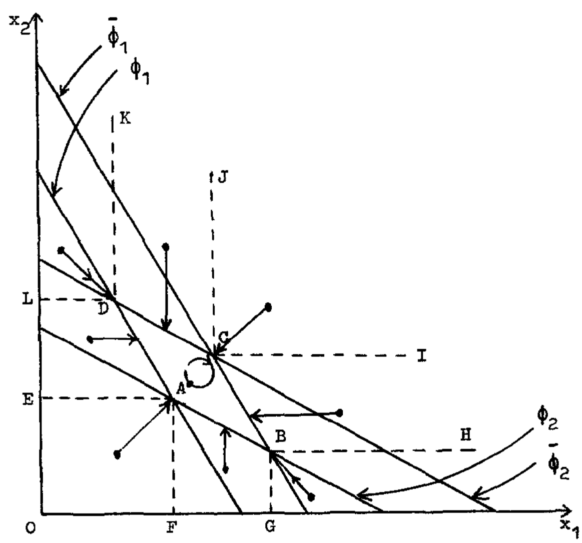

Fig. 4

regions given by the areas enclosed in EAF, FABG, GBH, HBCI, ICJ, JCDK, KDL, LDAE and ABCD. Given that x^1 is in each of these regions, what happens in substage 2 is shown by the arrows. For instance, if x^1 is in ABCD, $x_1^2 = x_2^2 = 0$. These are easy to check using the aggregate reaction function idea of fig. 3.

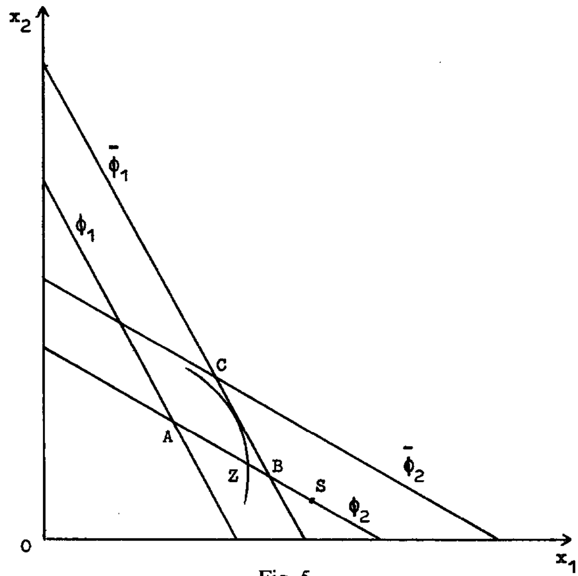

Fig. 5

It should be immediately obvious that the subgame perfect equilibria of the two-substage Cournot game can occur only on or within ABCD. This has an important implication. Suppose that the cost of production actually happened to be zero. That is, $c_i(x_i)=0$, for all x_i. Then ϕ_i and $\bar{\phi}_i$ would coincide. Hence the area ABCD would collapse to a point, namely the usual Cournot equilibrium point. It follows that the only subgame perfect equilibrium in such a model would be the Cournot outcome.

I shall here continue with the realistic assumption of the marginal cost of production being positive. Using the second-substage behaviour, summed up in fig. 4, it is obvious that point A (the Cournot outcome) is a subgame perfect equilibrium of G^*. I shall now isolate for each firm i, a subgame perfect equilibrium of G^* where i earns less profit than at A. Without loss of generality, I focus on firm 1.

Continuing with the informal exposition, let us isolate three points of significance in fig. 5. Let B be the point where firm 1's pseudo-reaction function intersects 2's reaction function. Let S be the Stackelberg outcome with 1 as leader. Next let C be the point of intersection between the pseudo-reaction functions of 1 and 2. Pick the iso-profit curve of firm 2 which is tangential to the line segment BC (i.e. the iso-profit curve which maximizes 2's profit and makes contact with 1's pseudo-reaction function between B and C). The point of intersection between this iso-profit curve and firm 2's reaction function is labelled Z. Let θ be an operator such that $X \equiv \theta(Z,S)$ means $X \in \{Z,S\}$ and $X_1 = \min\{Z_1, S_1\}$ where $X=(X_1,X_2)$, $Z=(Z_1,Z_2)$, and

$S=(S_1, S_2)$. In words, $\theta(Z, S)$ is that point among Z, S which lies furthest to the left. It is being claimed that $\theta(Z, S)$ is an output-pair that can be supported by subgame perfect strategies.[7] This is easily proved using the kind of argument employed in section 2.5 in conjunction with information about what happens in substage 2, which is summarised in fig. 4, and is, therefore, being omitted here.

It is obvious that at $\theta(Z, S)$, firm 1 earns a lower profit than at A. Hence $\theta(Z, S)$ can be used as a punishment for 1 in devising a trigger strategy. A similar argument can be constructed for firm 2 and it follows that collusion is possible even with free disposal.

5. Concluding remarks

The purpose of this paper was to explain collusion among perfectly rational oligopolists playing finite-duration games. One way of achieving this is to introduce some increasing returns in production or fixed costs. This could create a multiplicity of equilibria in the single-shot Cournot game and thus allow for collusive behaviour in the early stages of a supergame [Fraysse and Moreaux (1985); Harrington (1987)].

The present paper however uses a stage game which is itself an *extensive*-form game. The supergame therefore arises out of repetitions of an extensive-form game. One has to be careful in defining subgame perfection in this context because we have to consider subgames which arise after *n and a half* stage games. Collusion is then shown to arise under subgame perfection.

There was another objective in the paper. In the stage game considered here, the existing literature assumes that agents cannot dispose of goods once they are produced. This is clearly an unrealistic assumption. In section 4, therefore, it was assumed that disposal is free and it was shown that though this results in a shrinkage of the set of equilibria, the shrinkage is not enough to rule out collusion.

[7]In fact, all points on firm 2's reaction function between $\theta(Z, S)$ and A are supportable under subgame perfection. And, of course, a symmetric claim is true with the firms reversed.

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