Entry-Deterrence in Stackelberg Perfect Equilibria
Author(s): Kaushik Basu and Nirvikar Singh
Source: *International Economic Review*, Vol. 31, No. 1 (Feb., 1990), pp. 61-71
Published by: Wiley for the Economics Department of the University of Pennsylvania and Institute of Social and Economic Research, Osaka University
Stable URL: http://www.jstor.org/stable/2526628
Accessed: 16-09-2017 19:46 UTC

INTERNATIONAL ECONOMIC REVIEW
Vol. 31, No. 1, February 1990

ENTRY-DETERRENCE IN STACKELBERG PERFECT EQUILIBRIA*

BY KAUSHIK BASU AND NIRVIKAR SINGH[1]

This paper examines entry-deterrence in a duopoly where the post-entry game is Stackelberg. It is argued that in reality firms can use a broader range of precommitments than is allowed for in the literature. This paper permits such precommitments and analyses the perfect equilibria. It also allows for the fact that there may be fixed costs associated not only with entry but with beginning production. Several interesting possibilities are explained including the existence of excess capacity and the holding of inventories even in the absence of any uncertainty.

1. INTRODUCTION

As had been pointed out by Bain (1956) and Sylos-Labini (1962) and is well-known now, the behavior of a firm depends as much on its existing rivals as on the potential ones. This is the essence to analyzing barriers to entry. The present paper analyzes entry-deterrence in a duopoly, consisting of an incumbent and an entrant, where the post-entry game is Stackelberg with the incumbent playing leader. While a number of alternative characterisations of the post-entry game have been discussed in the literature (e.g., Cournot-Nash by Dixit 1980; Stackelberg with entrant as leader by Salop 1979), this particular characterisation has been ignored as uninteresting (Saloner, 1985, being an exception). And indeed it would be uninteresting if the standard cost-function (e.g., the kind used by Spence 1977; Dixit 1979, 1980) is used. In contrast, we consider a more realistic cost function in which fixed costs consist of two parts: *entry cost* and (production-) *commencement cost*. The former is the cost associated with entering an industry (acquiring a license, which in turn may require setting up an office, lawyer fees, etc.) and the latter is the usual cost of beginning positive production.

In this paper we also allow firms to go in for a broader ranger of commitments than is allowed for in the literature.[2] With these modifications the model of duopoly with the incumbent playing leader in the post-entry game (which is after all more natural than the entrant playing leader) becomes an interesting case explaining a large class of phenomena.

* Paper received December 1987; revised September 1988.

[1] Financial support from the UCSC Faculty Research Committee is gratefully acknowledged by Nirvikar Singh. A part of the work on this paper was done while Kaushik Basu was a Member at the Institute for Advanced Study, Princeton. The authors have benefited from comments received at a seminar at the Indian Statistical Institute, Delhi, and from two anonymous referees.

[2] A different direction to pursue would be to consider different *kinds* of capital and their role in entry-deterrence. See Eaton and Lipsey (1981) for an interesting analysis along these lines.

2. THE FRAMEWORK

Let x_1 and x_2 be the sales of firms 1 and 2 and let the inverse demand function be $p(x_1 + x_2)$, i.e., the firms produce a homogeneous good. We assume

(A1) p is thrice-differentiable, $p' \leq 0$ and there exists a real number n such that $p(n) = 0$.

We shall use 1 and 2 to represent, respectively the incumbent and the entrant firms. It is also useful to stress the difference between sales and production, because firms might want to build up stocks, and x_1 throughout represents firm i's sales, rather than output. The incumbent firm is able to precommit in two ways. These variables are denoted k_1 and r_1 respectively, and they enter the cost function of firm 1 as follows:

$$c_1(r_1, k_1, x_1) = f_1 + (v_1 - r_1)x_1 + r_1 \max\{k_1, x_1\}. \tag{1}$$

If r_1 were exogenous, this would be the standard cost function, (the one used by Dixit 1980; Bulow, Geanakoplos and Klemperer 1985; and others) with r_1 being the cost of capital and v_1 the per unit cost of all inputs. However, we allow firm 1 to choose r_1, to represent the fact that incumbent firms may precommit a given level of potential output (k_1) with varying degrees of readiness (r_1). This is a realistic assumption. The standard practice of treating r_1 as fixed seems unduly restrictive. After all, there is no reason why a firm cannot actually undertake a certain level of production and hold a part of it as stocks to threaten a potential entrant. This means that $r_1 = v_1$ is always open to firms.[3] We go further in treating r_1 as a variable which can take any value within $[0, v_1]$. Of course, we are measuring inputs in units of output.

One could get an intuitive picture of the kind of precommitment we are modeling by considering the marginal cost curve implied by (1). This is illustrated in Figure 1. If the firm precommits nothing ($r_1 = 0$, $k_1 = 0$) then the marginal cost curve is given by the line v_1D. If, however, $r_1 > 0$, $k_1 > 0$ (the case illustrated in the figure), then the shaded area is the cost that is precommitted (i.e., effectively this represents a "fixed" cost). Hence, the *effective* marginal cost curve is given by the line ABCD, which is exactly the case considered by Dixit, excepting for the fact that the height of the line segment AB is, in the present model, an endogenous variable. Note also that x_1 may exceed k_1. It would generally be the case that $f_1 > 0$, but in the case of firm 1 the existence of fixed costs makes no difference. So, for simplicity, we set $f_1 = 0$. Finally, note that the (piecewise) linear cost specification is restrictive, but we shall see that it is sufficiently rich to encompass a wide range of possible equilibria.

Next, firm 2's cost function is

$$c_2(x_2) = \begin{cases} 0 & \text{if } x_2 = 0, \\ f_2 + v_2 x_2 & \text{if } x_2 > 0. \end{cases} \tag{2}$$

[3] The case of inventory holding can be explained and interpreted in another way: It could be thought of as the case of committing the most extreme product-specific capital, namely, the product itself (see Ware, 1985).

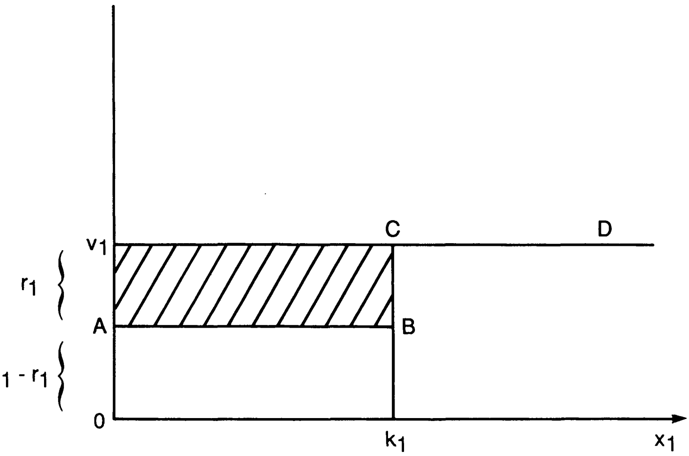

FIGURE 1

Hence we are interpreting the fixed cost, f_2, as a cost which is incurred only if firm 2 enters and produces. The sales of firm 2 are denoted by x_2 (but since for firm 2 sales and production never differ, x_2 may be treated as either). It would be possible to introduce fixed costs associated with entry itself. This is inessential: the crucial requirement is that there be some fixed cost associated with positive production.

A clarifying comment on our cost function: Using D as a variable which takes a value of 1 if firm 2 enters the industry and 0 if it does not, a more general cost function would be as follows:

$$\begin{aligned} c_2(x_2, D) &= 0, \text{ if } D = 0 \\ &= K, \text{ if } D = 1, \text{ if } x_2 = 0 \\ &= K + f_2 + v_2 x_2, \text{ if } x_2 > 0. \end{aligned}$$

(It is assumed throughout that $D = 0$ implies $x_2 = 0$.) Implicit in Dixit's and Spence's work is the assumption that $f_2 = 0$, but $K > 0$. Our paper, on the other hand, makes crucial use of the fact that $f_2 > 0$.[4] In fact, the case of the post-entry game being Stackelberg turns out to be interesting precisely because we do not assume away such fixed costs. In reality of course K would also be positive. This plays no role in our model and hence we set $K = 0$, which immediately reduces the cost function above to (2). So "entry" in this paper is a costless acquisition of a license to produce. The essential consequence of a positive *entry* cost (i.e., $K > 0$) is introduced in our paper by the simple lexicographic assumption that if a firm earns zero profit after entry, it would prefer not to enter. We could have, at the cost of some additional algebra, instead assumed that $K > 0$.

[4] We are grateful to Avinash Dixit for emphasizing to us the role of these two different kinds of fixed costs.

Given the above duopoly situation, we assume that firm 1 first chooses (r_1, k_1), its precommitment. Then firm 2 decides whether to enter or not, i.e., whether to set $D = 1$ or 0. After firm 2 chooses D, firm 1 selects its sales level x_1, followed by firm 2's choice of x_2. Hence an *outcome* of this sequence of choices is denoted by the quintuple (r_1, k_1, D, x_1, x_2). Since, in the post-entry game with $D = 1$, firm 1 chooses first, it is a Stackelberg leader and 2 a Stackelberg follower.

We introduce another assumption and some more notation before describing formally our equilibrium concept. A convenient simplification is that the potential entrant's output, given the incumbent's decisions, is unique. This has two parts, embodied in the following: Let $\Pi_2(x_1, x_2) = x_2 p(x_1 + x_2) - c_2(x_2)$, i.e., Π_2 (.) is firm 2's profit function.

(A2) (i) $\Pi_2(x_1, x_2)$ is strictly concave in x_2.
(ii) Firm 2 will not produce (enter) if it can at best earn the same profit by producing (entering) as by not producing (entering).

A2(i) ensures that firm 2's response to firm 1's sales is a function, say $R(x_1)$, except at the discontinuity created by $f_2 > 0$. By A2(ii), $R(x_1)$ is 0 at that level of x_1. It is easy to check that $R(x_1)$ is also 0 for greater x_1. We may define B_1 to be the smallest sales of firm 1 for which firm 2 produces nothing:

DEFINITION 1. $B_1 \equiv \min \{x_1 | R(x_1) = 0\}$.

We may also note that A2(i) may be derived from standard assumptions on the first two derivatives of the inverse demand (see e.g., Friedman, 1977).

In the next section, we begin analysis of the equilibrium outcomes of the duopoly situation above. The equilibrium notion we use is that of a perfect equilibrium. We therefore assume that at each move each firm chooses so as to maximize its profit, and takes into account that in the subsequent moves everyone will choose so as to maximize profits. An outcome which emerges from such a sequence of decisions will be referred to as a *Stackelberg perfect equilibrium*, or *SP-equilibrium* in brief.

3. BASIC RESULTS

In this and subsequent sections, we are chiefly concerned with the behavior of firm 1, since firm 2 is a Stackelberg follower. Hence we drop the '1' subscript for the incumbent where it is unambiguous. Firm 1's profit is described by

$$\Pi_1(r_1, k_1, D, x_1, x_2) = \begin{cases} p(x_1 + x_2)x_1 - c_1(r_1, k_1, x_1), & \text{if } D = 1 \\ p(x_1)x_1 - c_1(r_1, k_1, x_1), & \text{if } D = 0. \end{cases} \tag{3}$$

If $D = 1$, then $x_2 = R(x_1)$ and we define $\tilde{\Pi}_1(r_1, k_1, 1, x_1) \equiv \Pi_1(r_1, k_1, 1, x_1, R(x_1))$. Corresponding to A2(i), then, we have the following assumptions on Π_1:

(A3) (i) $\Pi_1(r_1, k_1, 0, x_1, 0)$ is strictly concave in x_1.
(ii) $\tilde{\Pi}_1(r_1, k_1, 1, x_1)$ is strictly concave in x_1.

A3(i) is again a standard assumption, but A3(ii) involves the third derivative of the inverse demand. A more explicit condition could be derived, but is not particularly insightful. A linear demand curve satisfies all these assumptions, for example (for the cost functions used here). In what follows, including the lemmas and theorems, A1, A2 and A3 are assumed throughout.

We now define different profit-maximizing levels of output for firm 1. These exist and are unique by our assumptions. In the following "argmax" denotes the maximizing value.

DEFINITION 2. $\phi(r) = \text{argmax}_x\ \theta(x, r)$ with $\theta(x, r) \equiv p(x)x - (v - r)x$.

Hence $\phi(r)$ denotes the output level that a monopolist would choose if his marginal cost of production was $(v - r)$ instead of v.

DEFINITION 3. $M(r, k) = \text{argmax}_x\ \Pi(r, k, 0, x, 0)$.

DEFINITION 4. $S(r, k) = \text{argmax}_x\ \tilde{\Pi}(r, k, 1, x)$.

$M(r, k)$ is the monopoly output of the incumbent firm with a commitment of (r, k), and $S(r, k)$ is its Stackelberg equilibrium output if entry occurs. The following lemma provides a characterization of monopoly output in the presence of precommitment.[5]

LEMMA 1. *For all* r in $[0, v]$ and all $k \geq 0$,

$$M(r, k) = \begin{cases} \phi(0), & \text{if } k < \phi(0) \\ \phi(r), & \text{if } k > \phi(r) \\ k, & \text{otherwise.} \end{cases}$$

REMARKS.

(i) $\phi(0)$ is, of course, the monopoly output or $M(0, 0)$.

(ii) Note that A3(i) implies that $\phi(r)$ is strictly increasing in r.

(iii) By Lemma 1 and (ii), $M(r, k)$ is nondecreasing in k. Reasoning entirely analogous to Lemma 1 and the above remarks yield exactly the same characterization for $S(r, k)$. In particular $S(r, k)$ is nondecreasing in k. This is shown in proving the next result, which is the main one of this section.

THEOREM 1. *If* $(r_1^*, k_1^*, D^*, x_1^*, x_2^*)$ *is an SP-equilibrium, then either* (i) $x_1^* = M_1(r_1^*, k_1^*)$ *and* $D^* = 0$ *(and, by implication,* $x_2^* = 0$*) or* (ii) $x_1^* = S_1(0, 0)$ *and* $x_2^* = R(x_1^*)$.

Theorem 1 may be illustrated. In Figure 2 AEBF is the reaction curve of firm 2. CC′ and DD′ are 1's reaction curves with $r = 0$ and $r = v$ respectively. Lemma 1 implies that CD is the segment within which the monopoly equilibria of firm 1 with

[5] Proofs of Lemma 1 and Theorem 1 are available in an appendix.

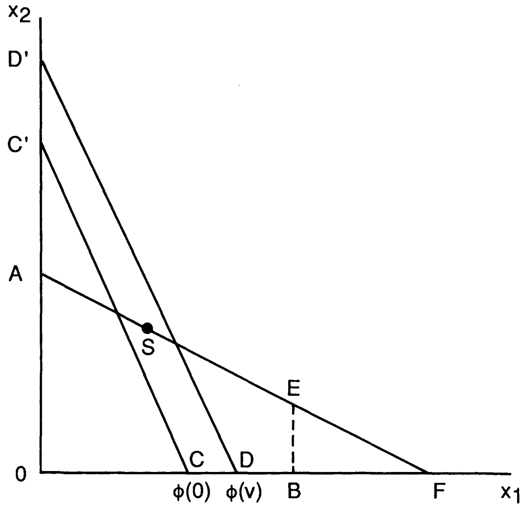

FIGURE 2

different commitment levels must lie. S is the usual Stackelberg point. Theorem 1 therefore asserts that the SP equilibrium must be either at S or on some point on CD. This theorem provides a partial characterization of SP-equilibria. Further properties of these equilibria are explored in the next section. Before going on to this, it is important to fully understand the significance of Theorem 1.

The intuitive argument behind Theorem 1 is clear: if entry is not to be deterred, then there is no point in making a costly commitment. Hence $x = S(0, 0)$. If entry is deterred, the firm is a monopolist, with the precommitment (r, k) necessary to deter entry. Hence, $x = M(r, k)$. This theorem is interesting for what it excludes. All configurations of outputs and strategies, apart from the ones just described, are ruled out as possible candidates for equilibria. By using this theorem we shall describe some special kinds of equilibria which can arise and which highlight the contrasts and similarities of our model and other works on entry-barriers including the pioneering works of Bain (1956) and Sylos-Labini (1962).

4. PROPERTIES OF STACKELBERG PERFECT EQUILIBRIA

An interesting property of SP-equilibrium, which highlights its contrasts with other models of duopoly, is a straightforward consequence of Theorem 1.

PROPERTY 1. *There exists a class of duopoly situations where* B_1 *is greater than* $M_1(0, 0)$, *and yet in the SP-equilibrium firm* 1 *produces its monopoly output,* $M_1(0, 0)$, *and* 2 *does not enter.*

This property is easy to see with Figure 3. It shows the usual reaction functions of firms 1 and 2 and the iso-profit curve of firm 1 which passes through the Stackelberg point S. Z_1 is the point where this iso-profit curve meets the x_1-axis.

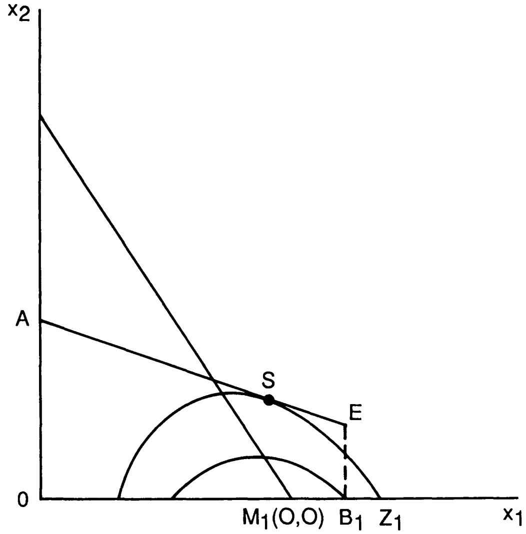

FIGURE 3

If $B_1 < Z_1$, as is the case in Figure 3, 1 will produce its monopoly output $M_1(0, 0)$ and 2 will not enter. This is because if 2 enters, 1 is best off producing B_1, which means 2 will produce nothing. Knowing this, 2 will not enter. And knowing that 2 will not enter, 1 chooses its best output, $M_1(0, 0)$. This contrasts sharply with Spence (1977) and Dixit (1979) where, given a situation as in Figure 3, firm 1 would produce B_1. The models of Bain and Sylos-Labini also suggest that 1 would produce B_1.

In a perfect equilibrium, on the contrary, the fact that the incumbent possesses a strategy to costlessly eliminate a new firm, should it enter, is enough to guarantee that the new firm will not enter. The incumbent does not have to *actually adopt* the elimination strategy (which, in Figure 3, entails producing B_1).

The crucial role played by the existence of commencement costs, that is, fixed costs associated with positive production rather than entry (see discussion in Sections 1 and 2), is worth emphasizing here. If there were no such costs, then if firm 1 produced $M_1(0, 0)$, firm 2 would enter. This is because once entry occurs it has no further fixed costs and so the break in its reaction function at B_1 in Figure 3 is no longer there. Knowing this, firm 1 would accommodate the entrant by moving to the Stackelberg equilibrium output S_1 (where S_1 is the projection of the point S on the horizontal axis). Since firm 2, in turn, knows this, it is in its interest to enter and settle for equilibrium at S.

It is easy to extend the example in Basu and Singh (1985) to demonstrate the next property.

PROPERTY 2. *There exist SP-equilibria where the incumbent firm produces more than it sells* ($r_1 = v_1$, $k_1 > x_1$).

Once the notion of commitment is broadened so as to make r a component of it, it immediately becomes clear that one form of commitment consists of actually undertaking production and holding inventories. This is a natural extension of the narrower idea of commitment used in the existing literature (Caves and Porter, Dixit, Spence, Ware). What this property asserts is that if the post-entry game is Stackelberg then in a perfect equilibrium it may be actually worthwhile for the incumbent to produce more than it sells. In Property 2, since $r_1 = v_1$, firm 1's production is equal to k_1, though sales equal x_1. In the early limit-pricing literature (e.g., Sylos-Labini) it is acknowledged that an incumbent might overproduce in order to discourage potential entrants. However, having overproduced it is not necessary to actually sell all the production. This possibility is ignored in the traditional oligopoly literature because sales and output were not always distinguished.

Usual explanations of inventory holding of firms are in terms of expected price fluctuations. Property 2 shows that the SP-equilibrium provides an alternative explanation of inventories.

PROPERTY 3. *There exist SP-equilibria where the incumbent firm chooses a level of readiness greater than zero, but leaves no unutilized commitment* ($0 < r_1 < v_1$, $k_1 = x_1$).

In this case the firm makes a commitment but the commitment provides no 'burden' because it is not left unutilized. At first sight the purpose of such commitment may seem unclear. But of course it is merely to ensure that cutting back production is costly to itself and thereby give a signal to the entrant that it will not be accommodating and cut back production should entry occur.

PROPERTY 4. *There exist SP-equilibria in which $B_1 > Z_1$, yet firm 2 does not enter.*

In Dixit (1979), as well as in Bain (1956), if $B_1 > Z_1$ then entry is ineffectively impeded and a usual Stackelberg equilibria is a *necessary* outcome. Property 4 suggests that in an SP-equilibrium this need not be so.

Properties 3 and 4 may be demonstrated geometrically. Figure 4 shows how to work out the iso-profit curves of a firm with commitment ($\hat{r}$, $\hat{k}$). Let $C'C$ and $D'D$ be firm 1's reaction with a commitment of 0 and $\hat{r}$ respectively. Let $\hat{k}$ be as shown in the diagram. Suppose *fbe* is an iso-profit curve corresponding to $C'C$ (i.e., with no commitment). Draw an iso-profit curve corresponding to $D'D$ (i.e., with a commitment of $r = \hat{r}$ and very large k, larger than g) passing through b. This is shown by *abg* in the figure. Then for a firm with commitment ($\hat{r}$, $\hat{k}$), an iso-profit curve is shown by *abe*. Other iso-profit curves may be similarly constructed.

Now we may demonstrate properties 3 and 4. Suppose in Figure 5, 1's iso-profit curve through B_1 intersects 2's reaction function at a point Q, to the left of Z_1, which is the point where the iso-profit curve through the Stackelberg point S touches the x_1-axis. Choose a point in between Z_1 and Q on the x_1-axis and call it T_1. For simplicity, assume 1's reaction function with $r_1 = v_1$ lies to the right of T.

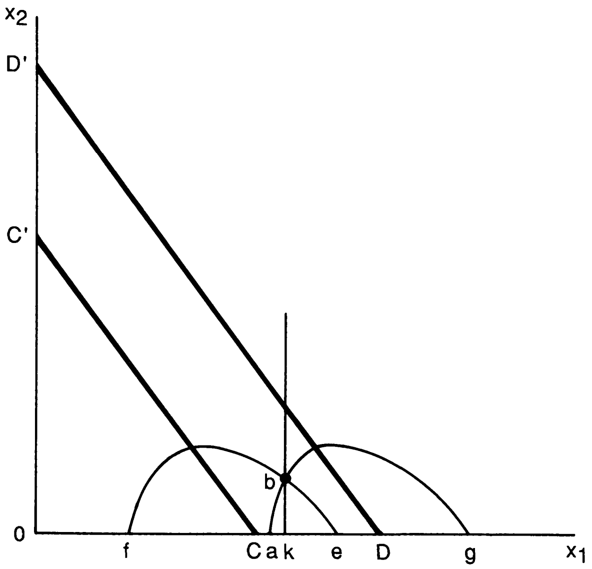

FIGURE 4

Such a reaction function and its associated iso-profit curves are shown by the broken lines. Suppose 1 sets $k_1 = T_1$ and $r_1 = v_1$. Then its iso-profit curves look like ADB_1, by the argument sketched above. Now if 2 enters, it will be best for 1 to produce B_1 (and be an iso-profit curve ADB_1). Knowing this 2 will not enter. Hence 1 can act as a monopolist once it has committed (v_1, T_1). By Lemma 1, his sales will be T_1. Therefore, 1 will be on a higher iso-profit curve than at S (namely

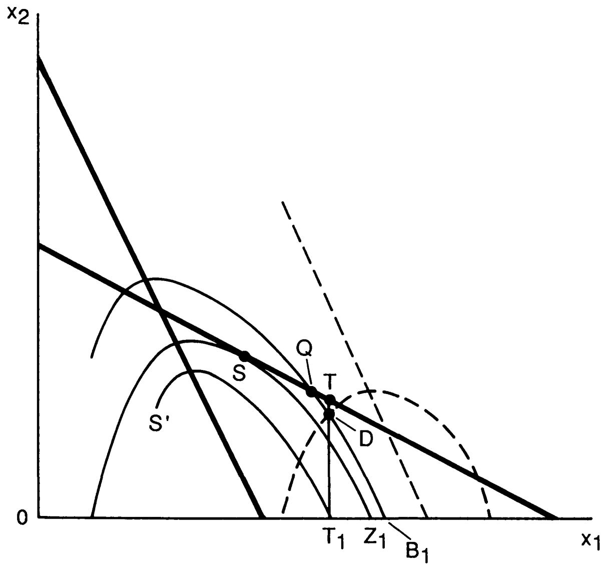

FIGURE 5

on curve T_1S'). This shows that there exists a strategy of firm 1 which dominates point S. Hence by Theorem 1, firm 2 will not enter and firm 1 will operate at $M_1(r_1, k_1)$ for some precommitment (r_1, k_1), thereby establishing both Property 3 and Property 4.

Delhi School of Economics, India, and Princeton University, U.S.A.
University of California, Santa Cruz, U.S.A.

APPENDIX

LEMMA 1. *For all r in $[0, v]$ and all $k \geq 0$,*

$$M(r, k) = \begin{cases} \phi(0), & \text{if } k < \phi(0) \\ \phi(r), & \text{if } k > \phi(r) \\ k, & \text{otherwise.} \end{cases}$$

PROOF. Suppose $k > \phi(r)$. For any $x > 0$,

$$\Pi(r, k, 0, \phi(r), 0) - \Pi(r, k, 0, x, 0) = [\theta(\phi(r), r) - rk] - [\theta(x, r) - r \max\{x, k\}]$$

$$= \theta(\phi(r), r) - \theta(x, r) + r[\max\{x, k\} - k] \geq 0,$$

since $\phi(r) = \operatorname{argmax}_x \theta(x, r)$. Hence $\phi(r) = M(r, k)$.

Suppose $k < \phi(0)$. For any $x \geq 0$,

$$\Pi(r, k, 0, \phi(0), 0) - \Pi(r, k, 0, x, 0) = \theta(\phi(0), 0) - \theta(x, 0) + r[\max\{x, k\} - x] \geq 0.$$

Hence $\phi(0) = M(r, k)$.

Finally, suppose $\phi(0) \leq k \leq \phi(r)$. If $x < k$, then $\Pi_x(r, k, 0, x, 0) = \theta_x(x, r)$. Since $\theta_x(\phi(r), r) = 0$ and $x < \phi(r)$, hence $\Pi_x(r, k, 0, x, 0) > 0$, by A3(i). If $x > k$, then $\Pi_x(r, k, 0, x, 0) = \theta_x(x, 0)$. Since $\theta_x(\phi(0), 0) = 0$ and $x > \phi(0)$, hence $\Pi_x(r, k, 0, x, 0) < 0$, by A3(i). Therefore, $M(r, k) = \operatorname{argmax}_x \Pi(r, k, 0, x, 0) = k$. Q.E.D.

THEOREM 1. *If $(r_1^*, k_1^*, D^*, x_1^*, x_2^*)$ is an SP-equilibrium, then either* (i) $x_1^* = M_1(r_1^*, k_1^*)$ *and* $D^* = 0$ *(and, by implication,* $x_2^* = 0$) *or* (ii) $x_1^* = S_1(0,0)$ *and* $x_2^* = R(x_1^*)$.

PROOF. As a first step, it will be proved that given any r,

$$k > k' \rightarrow S(r, k) \geq S(r, k'). \tag{4}$$

Suppose $k > k'$ and let $x' \equiv S(r, k')$. Definition 4 implies

$$\begin{aligned} &p(x' + R(x'))x' - (v - r)x' - r \max\{k', x'\} \\ &\quad > p(\hat{x} + R(\hat{x}))\hat{x} - (v - r)\hat{x} - r \max\{k', \hat{x}\}, \ \forall \hat{x} < x' \end{aligned} \tag{5}$$

The strict inequality in (5) is because $S(r, k')$ is unique by virtue of A3(ii). It is easily checked that if k' is replaced by k, inequality (5) remains unchanged. Thus even with k, firm 1 finds that x' earns a greater profit than all $\hat{x} < x'$. Hence as k' is replaced by k, firm 1 will not choose a smaller output, thereby establishing (4).

Let Q be the set of commitments of firm 1, given which, 2 prefers to stay out of the industry:

$$Q = \{(r, k) \mid S(r, k) \geq B_1\}.$$

First, consider the case where $(r^*, k^*) \in Q$. If $D = 1$, $x_2 = 0$ and firm 2 earns no profit. If $D = 0$, $x_1 = M(r^*, k^*)$ and 2 earns zero profit. By A2(ii), firm 2 chooses $D^* = 0$, and $x_1^* = M(r^*, k^*)$.

Next suppose $(r^*, k^*) \notin Q$. If $D = 1$, then $x_1 = S(r^*, k^*) < B_1$. Firm 2 earns positive profit. Hence $D^* = 1$, $x_1^* = S(r^*, k^*)$ and $x_2^* = R(x_1^*)$. Furthermore, $\tilde{\Pi}$ is decreasing in k for $k > x$. This, together with (4)—which implies that $(r^*, k) \notin Q$ for $k < k^*$—implies $k^* \leq x_1^*$. But then, $\tilde{\Pi}(r^*, k^*, 1, x_1^*) = p(x_1^* + R(x_1^*))x_1^* - v_1 x_1^* = \tilde{\Pi}(0, 0, 1, x_1^*)$. Suppose now that for some x_1,

$$\tilde{\Pi}(0, 0, 1, x_1) > \tilde{\Pi}(0, 0, 1, x_1^*). \tag{6}$$

Since $(0, 0) \notin Q$, (6) implies that firm 1 can do better by committing (0, 0), i.e., nothing. Hence, (r^*, k^*) cannot be part of a perfect equilibrium outcome. This contradiction establishes (6) as false. Thus $x^* = \text{argmax}_x\, \tilde{\Pi}(0, 0, 1, x) \equiv S_1(0, 0)$.

Q.E.D.

REFERENCES

BAIN, J. S., *Barriers to New Competition* (Cambridge: Harvard University Press, 1956).

BASU, K., AND N. SINGH, "Commitment and Entry-Deterrence in a Model of Duopoly," *Economics Letters* 18 (1985).

BULOW, J., J. GEANAKOPLOS AND P. KLEMPERER, "Holding Idle Capacity to Deter Entry," *Economic Journal* 95 (1985).

CAVES, R., AND M. PORTER, "From Entry Barriers to Mobility Barriers," *Quarterly Journal of Economics* 91 (1977).

DIXIT, A., "A Model of Duopoly Suggesting a Theory of Entry Barriers," *Bell Journal of Economics* 10 (1979).

———, "The Role of Investment in Entry-Deterrence," *Economic Journal* 90 (1980).

EATON, B. C., AND R. LIPSEY, "Capital, Commitment and Entry Equilibrium," *Bell Journal of Economics* 12 (1981).

FRIEDMAN, J., *Oligopoly and the Theory of Games* (Amsterdam: North Holland Publishing Company, 1977).

SALONER, G., "Excess Capacity as a Policing Device," *Economics Letters* 18 (1985).

SALOP, S. C., "Strategic Entry Deterrence," *American Economic Review*, Papers and Proceedings 69 (1979).

SPENCE, A. M., "Entry, Capacity, Investment and Oligopolistic Pricing," *Bell Journal of Economics* 8 (1977).

SYLOS-LABINI, P., *Oligopoly and Technical Papers* (Cambridge: Harvard University Press, 1962).

WARE, R., "Inventory Holding as a Strategic Weapon to Deter Entry," *Economica* 52 (1985).