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DUOPOLY EQUILIBRIA WHEN FIRMS CAN CHANGE THEIR DECISION ONCE

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Consider a duopoly in which each firm announces its production plan. If both approve, the plans get implemented. Otherwise the plans are rejected and the firms play the Cournot game. In this model each firm's perfect equilibrium output may be much smaller than in the usual Cournot model.

1. Introduction

Duopolists under Cournot equilibrium produce more than would be in their collusive interest. But collusion is not the only reason why duopolists may restrict output to less than would be predicted by the standard Cournot theory. If before playing the Cournot duopoly game the firms allow themselves one opportunity to 'collude' then the subgame perfect equilibrium of this extensive game may entail a much smaller production (and therefore a greater deviation from the efficient, competitive solution) than in the Cournot equilibrium. In fact, the production may be smaller than if obtained under effective collusion.

Saloner (1987) has recently analysed a duopoly where firms can 'change' their minds once. The model presented here could also be viewed as a variant of Saloner's model

2. The game

There are two firms producing the same good. In period 1, each firm writes down its proposed output level. That is, firm i, writes down a real number x_i^1. In period 2 each firm has to state whether it 'accepts' the scheme of production (x_1^1, x_2^1) suggested in period 1 or 'rejects' it.

If both accept, then (x_1^1, x_2^1) is put into effect. That is, firm i produces x_i^1 and earns a profit of

$$R_i(x_1^1, x_2^1) = p(x_1^1 + x_2^1)x_i^1 - c_i(x_i^1),$$

where $p(.)$ is the inverse demand function faced by the industry and $c_i(.)$ is firm i's total cost function.

If one or both firm rejects (x_1^1, x_2^1) then in period 3 each firm i again writes down its production plan, x_i^3, and these plans get implemented. Hence firm i earns a profit of $R_i(x_1^3, x_2^3)$.

Note that the game in period 3 is the standard Cournot one. This is the reason why in the introduction it was suggested that the model may be viewed as that of Cournot with one prior attempt at collusion.

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3. The perfect equilibria

The statement of the main theorem is easier if we make the following simplifying (but non-essential) assumptions. It will be maintained throughout that each firm's iso-profit function in the x_1x_2-space is concave and that there is a unique Cournot equilibrium. Let point N (see fig. 1) be the Cournot equilibrium and let R_{iN} be the iso-profit curve of i which gives i the profit it earns at N. Let M be the other point where R_{1N} and R_{2N} intersect. That is, if the two firms produce M_1 and M_2 they will earn the same profits as in the Cournot equilibrium. (We shall also use R_{iN} to denote the amount of profit earned by i at the Cournot equilibrium).

We may now state the central theorem of this note:

Theorem 1. In the 3-period model of duopoly described in section 2 the only points in the x_1x_2-space which can be supported under perfect equilibrium strategies are N and M.

Proof. To prove Theorem 1 consider stage 3 first: clearly in period 3 the game is different from the standard Cournot duopoly. Hence, if a rejection occurs in stage 2 the outcome will be N.

Let us now turn to stage 2. Suppose that in stage 1 (x_1^1, x_2^1) has been played. What is the equilibrium in the remaining subgame? It is evident that if $R_i(x_1^1, x_2^1) < R_{iN}$ then i will move 'reject' in period 2 because rejection guarantees that N will occur and he will earn R_{iN}. If, on the other hand, $R_i(x_1^1, x_2^1) \geq R_{iN}$, for $i = 1, 2$, then both players move 'accept', since 'rejection' is dominated and cannot therefore be part of a perfect equilibrium strategy.

In fig. 1, consider the set of all points were each player earns at least as much as at N. This is described by the area $MANBM$ (including the boundary). [1] So what we have shown is that if (x_1^1, x_2^1) lies outside the area $MANBM$ then it will be rejected. Otherwise it will be accepted.

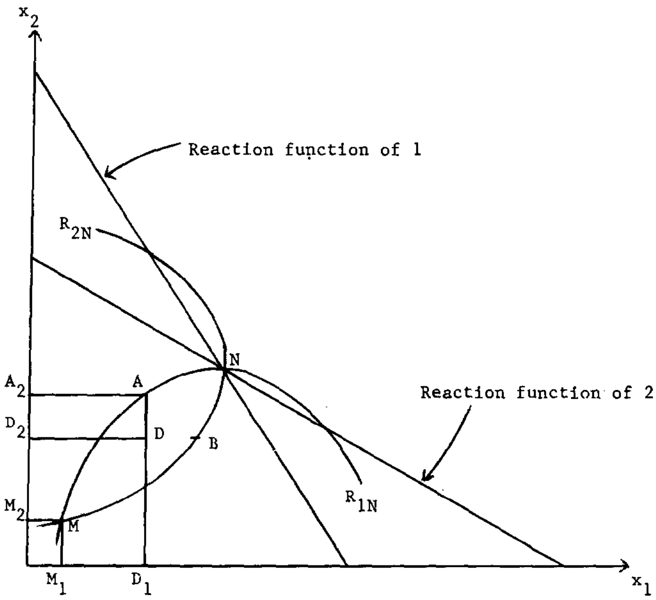

Fig. 1.

[1] Whenever we refer to the 'area $MANBM$' the reference will be to the *closed* set.

Finally, let us turn to the first period. From the above discussion it is clear that points outside the area *MANBM* cannot be supported by perfect equilibrium strategies. So let us turn to points in the area *MANBM*. Let us partition this area into the set of 'corner' points, $\{M, N\}$, and the set, Z, of 'non-corner' points (i.e., Z consists of all points in area *MANBM* excepting M and N). It will first be shown that no element of Z can be a perfect equilibrium.[2] Let D be an element of z (see fig. 1). Suppose in period 1 the two firms choose $(x_1^1, x_2^1) = (D_1, D_2)$. To see that this cannot be an equilibrium, note that for every point, E, in Z the following property is true:

Property (1). There exists a point in Z which is vertical above E or horizontally to the right of E.

Without loss of generality let us consider the point A which is in Z and vertically above D. Hence, if starting from D player 2 deviates to point A, then (we know from our analysis of periods 2 and 3) 2 would do better since A is an equilibrium in the subgame following (D_1, A_2) in period 1. Note that $R_2(D_1, A_2) > R_2(D_1, D_2)$. Therefore D cannot be a perfect equilibrium.

Since the points M and N do not satisfy Property (1) it is obvious that each of these points is a perfect equilibrium. □

4. A generalisation

Instead of permitting one attempt at collusion before resorting to the Cournot game as in the model of section 2, we could conceive of a more general game where several of such collusive attempts are permitted: Let n be any odd number. Consider the following n-period generalisation of the above 3-period game. In period 1 each of the two firms proposes an output level. If both accept this in period 2, it is implemented. Otherwise in period 3 each of the two firms again proposes an output. If both accept this in period 4, it is implemented. And so on. If by period n no production plan is accepted then whatever production plan, (x_1^n, x_2^n), is proposed in period n and gets implemented.

It is easy to see that no matter what the value of n is, the perfect equilibrium of the game consists of only points M and N in fig. 1.

References

Saloner, Garth, 1987, Cournot duopoly with two production periods, Journal of Economic Theory 42, 183–187.

[2] Distorting terminology a little I shall refer to a production plan as a perfect equilibrium if it can be *supported* by a perfect equilibrium strategy.