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The Right to Give up Rights

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Introduction

If we grant individuals rights over certain spheres of decision-making, it seems natural that we ought also to grant them the right to give up these rights. It has been argued that, if this principle is respected in social decision-making, then many of the fundamental paradoxes of liberty would disappear.

Beginning with Sen's (1970) work on the inconsistency between Paretianism and libertarianism, there have been many investigations in this area, and some have tried to show that our notion of liberty is, in itself, inconsistent (Gibbard, 1974; Farrell, 1976; Suzumura, 1978). In his widely discussed paper, Gibbard argued that many of these paradoxes would vanish if we granted individuals the right to give up rights and assumed that (α): an individual will waive his right if he expects to be "better-off" by doing so.[1] Let us refer to this escape route from the liberty paradox as the *meta-rights approach.*

By formalizing (α) in alternative ways, different special cases of the meta-rights approach can be explored. The best known of these special cases is Gibbard's own formalization. Gibbard assumed that an individual will waive his right over (x, y) if he prefers x to y and y to z and from x there exists a chain up to z, with the connections being made of the Pareto principle and other people's rights. This is stated rigorously in the next section and is referred to as the *Gibbardian waiver profile.*

This particular formalization of the meta-rights approach soon came under attack from Kelly (1976), who argued that the Gibbardian waiver profile is incentive-incompatible[2] in the sense that there exist situations where, by waiving rights in the way Gibbard suggests, individuals could actually harm themselves. So Kelly proposed an alternative formalization. Suzumura (1980), in turn, has shown that Kelly's approach runs into difficulties as well. Then, by examining other assumptions about situations where individuals would waive their rights, Suzumura has established a number of impossibility results.

One begins to suspect that there is some basic problem with the meta-rights approach itself. And indeed there is. This paper tries to show that all reasonable interpretations of the meta-rights approach are incentive-incompatible and, hopefully, it thereby obviates the need for analysing particular formalizations separately. More precisely, it is shown that, if individuals are allowed to waive their rights *voluntarily*, then there is no guarantee that they will do so in a way that resolves the liberty paradox. Thus Gibbard's theorems do not really bear out his own motivation.

The present work has another objective: to circumvent two rather pervasive shortcomings in the literature. The first one, noted by Sen (1976), is concerned with the interdependence of meta-rights. Clearly, there can arise situations where, if i waives his right, the reason for j to waive his right disappears.[3] This interdependence is not permitted in Gibbard and in much of the literature that followed. I quote Sen (1976, p. 224);

> The difficulty arises from each person deciding what right is "useless" for him on the basis of some presumption as to what rights the others would exercise, but, one person's decision not to exercise his right (on the supposed grounds of its being "useless" when others exercise their rights) renders erroneous another person's conviction that his right is "useless" (based on that person's assumption that others will exercise their rights). This problem of interdependence and of "correctable miscalculation" proves to be a deep one for the "pragmatic interpretation" of the Gibbard system.....

A second criticism stems from the fact that implicit in the Gibbardian waiver profile and other waiver profiles assumed by Kelly (1976) and Suzumura (1980) is the assumption that the social welfare function (SWF) is Paretian and libertarian. This follows from the fact that all these approaches look for chains using the Pareto criterion and liberty rights. Thus, if we are discussing an SWF that is libertarian and not necessarily Paretian, it is not clear why an individual would wish to waive his rights because there exists a chain from x to z linked by other people's rights *and the Pareto principle.* It is therefore an anomaly—not non-existent in the literature—to consider simultaneously a libertarian (and not necessarily Paretian) SWF and a Gibbard-type rights-waiving system. Similarly, if more properties are imposed on the SWF, then corresponding changes should be permitted in the waiver profiles. For example, if an SWF is libertarian, Paretian and also "equitable" (in some sense), then we should look for chains that may have some connections based on the equity criterion as well in deciding whether to waive a right or not.

Both these criticisms can be overcome by assuming that an individual's decision to waive his rights depends on how others waive their rights and on the nature of the SWF being used.

I. Rights and Meta-rights

Following Suzumura's (1980) notation and definitions, suppose that $N(\infty > \# N \geq 2)$ is the set of individuals. Given that X_i $(\infty > \# X_i \geq 2)$ is the set of characteristics of "personal" concern to $i \in N$, the set of social states is defined as[4]

$$X = \prod_{i \in N} X_i.$$

Let $\mathcal{R}$ be the set of all n-tuples of orderings (i.e. transitive, reflexive and complete binary relations) on X. $R \in \mathcal{R}$ implies $R = (R_1, \ldots, R_n)$ where R_i is the weak preference ordering of individual i. The asymmetric and symmetric parts of R_i are denoted by $P(R_i)$ and $I(R_i)$. Let D_i be the set of all pairs (x, y) where x and y are social states which differ only in their ith component.[5] Hence if (x, y) belongs to D_i, then the issue of choosing between x and y is personal to individual i. We therefore say that, if $(x, y) \in D_i$, then i has the *natural right* to decide between x and y;[6] and the n-tuple $(D_1, \ldots, D_n)$ is referred to as the *natural rights system.* When we say that an individual i has natural rights over $\{x, y\}$, we mean $(x, y) \in D_i$ and $(y, x) \in D_i$.

In keeping with Gibbard (1974), we allow individuals *meta-rights*, that is the right to waive their natural rights if they so wish. Let $S \subset X$. Then $D_i \cap S \times S$ are the pairs in S over which i has natural rights. Let W_i^S be the pairs over which he waives his natural rights in S. Hence $W_i^S \subset D_i \cap (S \times S)$. Let $\mathcal{S}$ be the collection of all non-empty subsets of X. A *waiver profile*, W, is defined as follows:

$$W \equiv \{W^S\}_{S \in \mathscr{S}} \equiv \{W_i^S\}_{S \in \mathscr{S},\, i \in N}.$$

A *social choice rule* (SCR) or a "voting scheme" is a function C, which, for all non-empty sets of alternatives S, for all n-tuples of preference orderings R and for all waiver profiles W, specifies a subset of S containing no more than one element. Thus, using $\mathscr{W}$ to denote the set of all waiver profiles, we have

$$C: \mathscr{S} \times \mathscr{R} \times \mathscr{W} \to \{T \subset X \mid \# T \leq 1\}.$$

If, given a waiver profile W, it is true that for all S, for all R, C is non-empty, then the SCR is said to be *decisive* (given W).

The two central ethical axioms may now be stated. The Pareto principle says that if everybody prefers x to y, y must not be chosen in the presence of x.

Axiom P The SCR, $C(\cdot)$, satisfies axiom P iff

$$[(x, y) \in \bigcap_{i \in N} P(R_i)] \to [x \in S \to y \notin C(S, R, W)].$$

The liberty axiom is stated directly introducing the idea of meta-rights. It says that, if an individual has natural rights over (x, y) which he has not waived, then, if he prefers x to y, y must not be chosen in the presence of x.

Axiom L The SCR, $C(\cdot)$, satisfies axiom L iff

$$[\text{for some } i \in N,\ (x, y) \in P(R_i) \cap D_i \setminus W_i^S] \to [x \in S \to y \notin C(S, R, W)].$$

In most of the existing literature an individual's decision to waive his rights depends neither on the SCR nor on how others waive their rights. It depends only on the preference profile, R, in question. Both the pervasive shortcomings in the literature mentioned in the introduction section can be shown to originate from this restrictive assumption. Consequently, in this paper we shall permit individuals to base their decision to waive rights on the nature of the SCR, the preference n-tuple in question, and also on how others exercise their meta-rights. Hence,

$$\text{(1)} \qquad \text{given } C(\cdot) \text{ and } R, \qquad W_i^S = W_i^S(W_1^S, \ldots, W_{i-1}^S, W_{i+1}^S, \ldots, W_{\# N}^W) \ldots$$

If individuals have no meta-rights, i.e. if $W_i^S = \phi$ for all S and for all i, then Gibbard has shown that, let alone P, there is no decisive SCR satisfying L. He identified the absence of meta-rights as the principal culprit in the original liberty paradox (Sen, 1970). He went on to argue that individuals ought to have the right to give up rights and that that would remove the conflict between the principles of Pareto and liberty. Clearly, the resolution of the paradox depends on how individuals exercise their meta-rights, i.e. on the specification of (1). Let us first consider Gibbard's own assumption.

The *Gibbardian waiver profile* (GWP), $\hat{W}$, is defined as follows:

$$(y_r, y) \in \hat{W}_i^S \text{ iff there exists } y_1, \ldots, y_{r-1} \text{ in } S \text{ such that } (y, y_1) \in R_i, y \neq y_1$$

and

$$(2) \qquad \forall t \in \{1, \ldots, r-1\}\colon (y_t, y_{t+1}) \in \left[\bigcap_{j \in N} P(R_j)\right] \cup \left[\bigcup_{j = N\setminus\{i\}} \{D_j \cap P(R_j)\}\right].$$

Gibbard proved that, given a GWP, there exists a decisive SCR satisfying axioms P and L. The attractiveness of this theorem depends on how the GWP is interpreted. Consider the following description by Suzumura (1980, p. 411) of the intuitive appeal of Gibbard's specification:

> An ingenious proposal crystallized in Gibbard's third libertarian claim [i.e. axiom L based on the waiver profile, $\hat{W}$] is to make individual's libertarian rights *alienable* in the cases where *the exercise of one's libertarian rights brings him into a situation he likes no better than the situation that would otherwise have been brought about.* [my italics]

If this is where the intuitive appeal of the meta-rights approach lies—and this is indeed where it should lie—then, unfortunately, Gibbard's theorem lacks intuitive appeal. However, instead of establishing this specifically for the Gibbardian waiver profile, I shall prove a more general theorem of which this is a corollary.

To motivate this, note that the new *approach* opened up by Gibbard, what I call here the meta-rights approach, is broader than Gibbard's *theorem*. After all, the theorem uses one particular formalization of (1), namely the Gibbardian waiver profile. So perhaps there are other specifications of (1) that are incentive-compatible. And anyway, the Gibbardian waiver profile does have important weaknesses.

First of all, as noted by Kelly (1976) and also by Suzumura (1980, p. 413), there may exist a "sequence $\{z_1, z_2, \ldots, z_{\lambda^*}\}$ which seems to repair in the eyes of the individual i the damage caused upon him by a sequence $\{y_1, y_2, \ldots, y_\lambda\}$." In that case the individual would not waive his right, unless there is yet another sequence that nullifies the effect of $\{z_1, z_2, \ldots, z_{\lambda^*}\}$.

Even with this correction, one difficulty remains (and this criticism applies to Kelly, 1976 and Suzumura, 1980, as well). Note that the GWP is characterized by a presumption on the part of each individual that others never waive their rights, because otherwise (2) would have to read

$$\forall t \in \{1, \ldots, r-1\}\colon (y_t, y_{t+1}) \in \left[\bigcap_{j \in N} P(R_j)\right] \cup \left[\bigcup_{j \in N\setminus\{i\}} \{(D_j \setminus W_j^S) \cap P(R_j)\}\right].$$

In other words, the functions (1) are constant mappings in the GWP and also in the waiver profiles defined and discussed by Kelly and Suzumura. This difficulty, as mentioned above, has been noted by Sen (1976).

To rectify this, we have to look for a specification of (1) that allows for interdependence in waiver sets. But the next result is a general one, which shows that all such effort is bound to be futile because no matter how (1) is specified, a resolution between axioms P and L necessarily entails involuntary rights-waiving, in some sense. The definition and theorem that follow establish this.

A matter of notation first. Given a waiver profile W, define $W_{\sim iS}$ as a waiver profile that differs from W (if at all) only in the (i, S)th element. In other words, if $W_{\sim iS} = \bar{W}$, then $\bar{W}_j^K = W_j^K$, for all $(j, K) \neq (i, S)$.

Definition 1 Given an SCR $C(\cdot)$, and a waiver profile W, individual i is subject to *involuntary rights-waiving* if it is *not* the case that for all S and R,

(i) for all $W_{\sim iS}$, $C(S, R, W_{\sim iS})$ is non-empty, and
(ii) $(x, y) \in R_i$ where $\{y\} = C(S, R, W_{\sim iS})$ and $\{x\} = C(S, R, W)$.

Theorem. For all decisive SCRs satisfying P and L, given any waiver profile, W, there must exist an individual who is subject to involuntary rights-waiving.

Proof Given that $x_i \in X_i$ and $x = x_3, \ldots, x_n$ and $x' = x_3, \ldots, x_n$, let a, b, c and d be distinct social states defined as follows:

$$a = (x_1, x_2, \hat{x})$$
$$b = (\tilde{x}_1, x_2, \hat{x})$$
$$c = (x_1, x_2, \hat{x}')$$
$$d = (x_1, \tilde{x}_2, \hat{x}').$$

Note that $(a, b) \in D_1$ and $(c, d) \in D_2$. Let $K = \{a, b, c, d\}$ and $R \in \mathscr{R}$ be such that its restriction on K is as follows (in descending order of preference from left to right):

$$R_1|K: c\,a\,b\,d$$
$$\forall j \in N\backslash\{1\},\ R_j|K: b\,d\,c\,a.$$

Any waiver profile in K must be one of the following four kinds:

Case 1 $\quad W_1^K = \{a, b\},$[7] $\quad W_2^K = \{c, d\}$.

Case II $\quad W_1^K = \phi, \quad W_2^K = \phi$.

Case III $\quad W_1^K = \{a, b\}, \quad W_2^K = \phi$.

Case IV $\quad W_1^K = \phi, \quad W_2^K = \{c, d\}$.

Case I, it is easy to check, is the Gibbardian waiver profile. Let us consider this case first. Note that by axiom P,

$$d, a \notin C(K, R, W).$$

Since both 1 and 2 have waived their respective rights in K, axiom l is ineffective. hence, if the SCR satisfying axioms P and L is decisive, then

$$\text{either } \{b\} = C(K, R, W) \quad \text{or } \{c\} = C(K, R, W).$$

Without loss of generality, assume the former. Define $W_{\sim 1K}$ by replacing W_1^K with $\bar{W}_1^K$, where $\bar{W}_1^K = \phi$. Then either

$$\{c\} = C(K, R, W_{\sim 1K}) \quad \text{or } \phi = C(K, R, W_{\sim 1K})$$

since b is now ruled out by the exercising of 1's rights. Hence individual 1 is subject to involuntary rights-waiving.

Case II is one where no rights are waived. This is the same situation as in the original Sen (1970) paradox: by axiom P, a and d cannot be chosen. By axiom L, b and c cannot be chosen. Hence $C(K, R, W) = \phi$. Thus the SCR is not decisive.

Now consider case III. Here a and d are ruled out by axiom P, and c is ruled out by axiom L. Hence, given a decisive SCR, it must be that $C(K, R, W) = \{b\}$. But $C(K, R, W_{\sim 1K}) = \phi$, where $W_{\sim 1K}$ is got by replacing W_1^K with $\bar{W}_1^K \equiv \phi$, for the same reason as in case II. Hence individual 2's rights-waiving described in case III does not dominate all other rights-waiving schemes open to him. So he is subject to involuntary rights-waiving.

Case IV is symmetric to case III. Q.E.D.

It is interesting to note that in case I, which is the Gibbardian case, we showed that each individual's specified waiver set is not dominant by comparing it with a case where he does not waive any of his rights. Hence, not only have we shown that Gibbardian rights-waiving is non-optimal, but we have shown, in particular, that it entails forcing people to forgo their rights.

II. Involuntariness and Equilibrium

It is important to appreciate that definition (1) is *one* formalization of the intuitive notion of involuntariness. There can be others. To see this it is useful to visualize the above problem in game-theoretic terms. Suppose society is about to choose from a set S, which is a subset of X. The SCR and the preference profile R are known. Individual i's problem is to choose a waiver set W_i^S in order to do as well as possible in terms of his preference relation. This is however, not a well-defined game, because the choice set corresponding to some strategy n-tuples, $\{W_i^S\}_{i \in N}$, may be empty. And since it is reasonable to suppose that a person's preference over $\{x\}$ and ϕ—in brief, over $\{x, \phi\}$—is incomplete, it is possible to interpret the remark that "given $\hat{W}_i^S$ for all $i \neq j$, $\hat{W}_j^S$ is j's best strategy" in different ways. Two reasonable interpretations are: (i) "there does not exist $\bar{W}_1^S$ such that the chosen social state corresponding to $(\hat{W}_1^S, \ldots, \hat{W}_{j-1}^S, \bar{W}_j^S, \hat{W}_{j+1}^S, \ldots, \hat{W}_n^S)$, where n denotes $\# N$, is preferred by j to the social state corresponding to $\{\hat{W}_i^S\}_{i \in N}$"; (ii) "the social state chosen, given, $\{\hat{W}_i^S\}_{i \in N}$, is considered by j to be at least as good as any of the social states that would be chosen if he changed his $\hat{W}_j^S$ to some other waiver set." Since his preference over $\{x, \phi\}$, for any $x \in X$, is incomplete, this interpretation requires that the social choice set be non-empty for all strategy n-tuples $(\hat{W}_1^S, \ldots, \hat{W}_{j-1}^S, W_j^S, \hat{W}_{j+1}^S, \ldots, \hat{W}_n^S)$ that we can get by varying W_j^S.

If the pay-off matrix was *well-defined*, meaning thereby that the social choice set was never empty, the two interpretations, (i) and (ii), would coincide. It is not difficult to see that in this paper the definition of involuntary rights-waiving is based on interpretation (ii). That is, a person is involved in involuntary rights-waiving if his waiver set does not satisfy optimality in the sense of (ii). (It should be clear at this point why we assumed that $\# C(S, R, W) \leq 1$. Otherwise, to define a person's "best" waiver set, we would need to extend his preference ordering on X to a preference ordering on the power set of X; and this—as is becoming increasingly clear—can be a formidable task.)

Using this game-theoretic approach, the theorem of Section I may be stated as an impossibility result. Given an SCR and a preference profile, let $\{\hat{W}_i\}_{i \in N}$ be described as *Nash-consistent* over S if there does not exist any individual who is subject to involuntary rights-waiving over S (i.e. if (ii) is true for all $j \in N$). The notion of Nash consistency is similar to the well-known concept of Nash equilibrium and would be exactly the same if the pay-off matrix were well-defined. Now we are in a position to restate our theorem: there does not exist any decisive SCR or waiver profile that satisfies axioms P and L and Nash consistency on every $S \subset X$.

While I have used a Nash-type equilibrium concept, and have, therefore, assumed that each person supposes that others do not respond to his change of strategies, it is possible to use other solution concepts where individuals have "conjectures" about the response of others. I do not however pursue this line here.

Once our theorem is restated in terms of Nash consistency, it acquires a certain similarity to Sen's (1983, p. 26) claim that

> in a situation exemplifying the conflict between the Pareto principle and individual liberty, there might exist no equilibrium at all—with some states being rejected by the Pareto-improving contract and the others being rejected by individual decisions over their personal spheres. . . . The impossibility of the Paretian liberal—interpreted in terms of descriptive choice—leads to a game with an empty "core".

While this result and the theorem in this paper are both about the non-existence of equilibrium, it is important to distinguish between the two. Sen's equilibrium is a situation in which, among other things, a pareto-improving contract is not possible. Thus a Pareto sub-optimal state cannot qualify as an equilibrium. In our framework, individuals act atomistically and the search is for a Nash-type equilibrium, which of course may be Pareto sub-optimal (as is the case with the Nash–Cournot equilibrium in the standard duopoly model). What we establish is that even such a Pareto sub-optimal equilibrium does not exist.

Finally, a comment on this paper's exclusive focus on the pragmatic interpretation of Gibbard (see note 1). What is interesting is the interconnection between the pragmatic and ethical interpretations. As Sen (1976) defines it, the ethical interpretation is concerned with how the conflict—between Pareto and liberty—*ought* to be resolved via rights-waiving. The pragmatic approach, on the other hand, examines whether, once people are free to give up their rights, the conflict continues to exist or not. Thus, an attempt to solve the pragmatic problem is equivalent to trying to determine whether or not the ethical problem exists at all. To that extent, this paper does have some bearing on the ethical question. What it shows is that the ethical problem remains given the Gibbardian system of rights-waiving; more importantly, it will remain no matter what system of voluntary rights-waiving is considered.

III. Persistence of the Original Paradox

This concluding section draws attention to an apparently perplexing feature of the meta-rights approach, concerning the "strength" of axioms.

In most areas of social choice, as we impose additional axioms on the social welfare function (e.g. the Pareto axiom, the non-dictatorship axiom, etc.) we, for quite obvious reasons, move closer to an impossibility result. In the approach adopted in this paper this need not be so. Consider an SCR satisfying axiom L. It has been argued above that it is reasonable to allow each individual's waiver profile to depend on, among the other things, the nature of the SCR. For example, a Nash-consistent waiver profile is quite clearly characterized by this kind of interdependency. In this framework it is not clear that additional axioms move us closer to an impossibility result. This is for the interesting reason that, with the imposition of each new axiom on the SCR, the waiver profile gets altered and this in turn changes the strength of the liberty axiom. Thus in this area it is misleading to make a remark like "Axiom L itself gives us an impossibility result, without having to invoke any other axiom (e.g. P)."

In this section I explore a related issue which also clarifies further the relation between Sen's (1970) original framework and that of Gibbard (1974).

Consider "weakening" the liberty axiom to "minimal libertarianism" in the sense of Sen (1970). That is, suppose that only two individuals have natural rights and that too over one pair each. In particular, assume that 1 has rights over $\{x, y\}$ and 2 has rights over $\{z, w\}$, and assume that x, y, z and w are distinct elements.[8] The *axiom of minimal liberty* is the same as axiom L but is based on this restricted domain of natural rights. That is, if individual 1 does not waive his right over $\{x, y\}$, then he is decisive over these elements; and similarly for individual 2.

What is interesting is that this "weakening" of the liberty axiom strengthens the impossibility result implied by our theorem. Note that ours is an "existential" result: it asserts that there exists a problem in the form of involuntary rights-waiving *somewhere* in the domain of the SCR. If we now replace axiom L by the axiom of minimal liberty, then it turns out that in *every* situation where the Sen paradox is rescued by Gibbardian rights-waiving there must exist involuntary waiving of rights.

Let $\hat{D}_1 = \{(x, y), (y, x)\}$ and $\hat{D}_2 = \{(w, z), (z, w)\}$; i.e., $\hat{D}_1$ and $\hat{D}_2$ is the natural rights system. Recall that Sen's original paradox arises if there exists a sequence $(z_1, , \ldots, z_r)$ in X such that

$$(3) \qquad (z_t, z_{t+1}) \in \left[\bigcap_{i \in N} P(R_i) \right] \cup [\hat{D}_1 \cap P(R_1)] \cup [\hat{D}_2 \cap P(R_2)].$$

where t belongs to a modular number system (mod r).[9] In the ensuing discussion all the subscripts t, t', t'' should be treated as belonging to such a number system.

When R is such that a cycle of the above kind occurs, we shall refer to it as a *potential paradox situation.* Gibbard showed that, once individuals are allowed to waive their rights and given that they do so in a particular way (namely, as specified by the Gibbardian waiver profile), the paradox vanishes.

Assume that given a certain $R \in \mathscr{R}$, $(z_1, \ldots, z_r)$ constitutes a potential paradox situation. This implies that there must exist t' and t'' such that

$$(4) \qquad (z_{t'}, z_{t'+1}) \in \hat{D}_1 \cap P(R_1)$$

and

$$(5) \qquad (z_{t''}, z_{t''+1}) \in \hat{D}_2 \cap P(R_2).$$

In other words, at least two steps in the cycle must be based on rights. If not, then at least one individual's preference must be intransitive. This is easy to see. Suppose (5) is false. Then (3) and (4) implies that, for all t, $(z_t, z_{t+1}) \in P(R_1)$, which implies intransitivity in 1's preference.

So (4) and (5) must be true. Since x, y, z, w are distinct, neither $z_{t'+1} = z_{t''}$ nor $z_{t''+1} = z_{t'}$. This implies, given (3), that

$$(z_{t'+1}, z_{t'+2}) \in \bigcap_{i \in N} P(R_i)$$

and

$$(z_{t''+1}), z_{t''+2}) \in \bigcap_{i \in N} P(R_i).$$

From this it follows, given Gibbardian rights-waiving, that both individuals will waive their rights.[10] Then, given a decisive SCR satisfying the Pareto criterion and the liberty criterion, the only elements that can be chosen from the set $\{z_1, \ldots, z_r\}$ are $z_{t'+1}$ and $z_{t''+1}$. This is because by the Pareto criterion all other elements are ruled out, and since all rights have been waived, the liberty axiom is inconsequential.

Without loss of generality assume that $z_{t'+1}$ is the chosen element.[11] If individual 1 had not waived his right, then $z_{t'+1}$ could not have been chosen. Hence the chosen set would have to be $\{z_{t''+1}\}$ or ϕ. Since individual 2 has rights over $(z_{t''}, z_{t''+1})$, the sequence from $z_{t''+1}$ to $z_{t'+1}$, i.e. $z_{t''+1}, z_{t''+2}, \ldots, z_{t'}, z_{t'+1}$, must be linked by individual 1's rights or the Pareto criterion. Hence by the transitivity of individual 1's preference $(z_{t''+1}, z_{t'+1}) \in P(R_1)$. Hence, either (i) or (ii) of definition (1) is violated. Since we had started from an arbitrary potential paradox situation, this establishes the disturbing proposition that, given minimal libertarianism, in *every* situation that is a potential paradox situation, the Gibbardian solution *necessarily* involves involuntary rights-waiving.

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NOTES

[1] In this paper, attention is restricted to a "pragmatic" interpretation of the "alienable rights system" as opposed to an "ethical" one (see Sen, 1976).

[2] This is not meant in the sense of strategic manipulation as in Karni (1978).

[3] While this paper alows for strategic behaviour, it should not be confused with Gardner's (1980) analysis of strategic consistency. In Gardner's paper, an agent goes in for strategic behaviour regarding which preference to reveal,—the real one or a false one. In the present paper, it is assumed that real preferences are known. Strategic behaviour comes in at the stage of meta-rights, i.e. in the decision as to whether to waive a right or not. There is no "dishonesty" problem here. Thus the issues raised in this paper would persist even if everybody's mind were known. To that extent the present paper, though explicitly concerned about pragmatic issues, highlights normative difficulties as well.

[4] Of course social states may differ in "non-personal" characteristics as well, and in a more complete description these ought to be brought in (see Suzumura, 1980), but for the present purpose it is adequate to treat all "non-personal" characteristics as constant and therefore irrelevant.

[5] Formally, $D_i = \{(x, y) \in X \times X | x_j = y_j, \text{ for all } j \neq i\}$.

[6] Note that $(x, y) \in D_i$ implies $(y, x) \in D_i$.

[7] This should be more correctly written as $W_I^K = \{(a, b), (b, a)\}$, since rights over pairs are always symmetric.

[8] We may ignore here the fact that social states are elements of the Cartesian product of personal characteristics, and treat them instead as primitives.

[9] A modular number system (mod r) consists of r distinct numbers $1, \ldots, r$. All other integers are treated as equal to one of these numbers by the following rule. Let n be any integer and let $n = a + kr$ where $a \in \{1, \ldots, r\}$ and k is an integer (note that for each n there is a unique such a). Then we say $n = a$, or more correctly $n = a$ (mod r). This number system has the advantage that $r + 1 = 1$. Hence, when we want to refer to (x_r, x_1), we may write, instead, (x_r, x_{r+1}). This obviously simplifies notation when discussing cycles.

[10] This is proved as follows. Consider the sequence from $z_{t'+2}$ to $z_{t'}$; i.e.,

$$z_{t'+2} z_{t'+3} \cdots z_{t'-1} z_{t'}.$$

From (3) it follows that each adjacent pair in this belongs to either of the three:

$$\text{(i)} \bigcap_{i \in N} P(R_i). \quad \text{(ii)} \hat{D}_1 \bigcap P(R_1). \quad \text{(iii)} \hat{D}_2 \bigcap P(R_2).$$

(iii) is ruled out by (4) and by the fact that each individual has rights over only one pair. Hence if 1 claims his right, then by the Pareto axiom and individual 2's rights we can go all the way up to $z_{t'+2}$. But $(z_{t'+1}, z_{t'+2}) \in P(R_1)$. Hence, in accordance with the definition of the GWP, individual 1 waives his rights over $(z_{t'}, z_{t'+1})$. A similar proof is possible for 2.

[11] A symmetric proof of involuntary rights-waiving on the part of individual 2 is possible if the chosen element is $z_{t''+1}$.

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