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OPTIMAL POLICIES IN DUAL ECONOMIES*

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I. INTRODUCTION

A dual economy is typically characterized as one that has an urban industrialized sector with institutionally fixed wages and a rural sector [Lewis, 1954]. Such economies and their optimal shadow wages have been analyzed a great deal (see, e.g., Sen [1968]). But more recently, Todaro [1969] and Harris and Todaro [1970] have given a new dimension to the problem with the introduction of a migration equilibrium condition (see also Fields [1975], and Mazumdar [1976]). Harris and Todaro (HT) argued that in a migration model, such as theirs, no single wage subsidy can ensure optimality. But Bhagwati and Srinivasan [1974] have elegantly demonstrated that there does exist a wage subsidy S^*, which, if given to all sectors, leads the economy to optimality.

A serious problem with the Bhagwati-Srinivasan (BS) optimal subsidy is that a particular component of their subsidy formula is the marginal product of labor *in the optimal situation.* To suppose that this will be known to the government at the time when the subsidy is given (i.e., prior to attaining the optimum) is an extremely strong informational assumption.[1]

Fortunately, however, the BS result can be substantially generalized. I shall argue that in the HT model there is a whole interval of optimal subsidies, and S^* is only one element in this interval. While S^* guarantees optimality, so does any other subsidy in this interval. It is then shown that the prevailing urban market wage is an element of this interval. Hence, the government can now give a subsidy which ensures optimality, the value of which is easily determined. Similar results hold for the HT second-best subsidy.

The next two sections outline the HT model and develop a simple diagram that illustrates the complete migration equilibrium. The diagram is an extremely useful aid for the analysis of economic policy.

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1. In fact, recent studies of decentralized iterative planning are largely motivated by the need to avoid such strong informational assumptions. Another way of avoiding this is to assume a partial-equilibrium model where the sector that the government can control is relatively small (e.g., Sen[1968]).

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II. The Harris-Todaro Model

There are two sectors: the rural (R) and the urban or modern (M). They produce X_R and X_M units of output and employ L_R and L_M units of labor. The output in each sector is a function of labor.[2]

$$X_R = f_R(L_R); \quad f_R' > 0, f_R'' < 0 \tag{1}$$

$$X_M = f_M(L_M); \quad f_M' > 0, f_M'' < 0. \tag{2}$$

Labor availability is fixed and, by assumption, equal to unity. Hence,

$$L_R + L_M \leq 1; \quad L_R, L_M \geq 0. \tag{3}$$

I shall assume, for simplicity, that both sectors produce the same good (though by different techniques). This leaves all the major HT and BS results unaltered. This assumption can be viewed in an alternative manner. We could think of X_R and X_M as different commodities in an open economy. Then, assuming constant world prices, we may redefine the units of X_R and X_M such that both their prices are equal to unity.

As in the HT model, we assume that the social welfare function is $U = U(X_R, X_M)$, $U_1 > 0$, $U_2 > 0$. But since in the present model X_R and X_M refer to the same commodity, the social welfare function may be simply written as $U = U(X_R + X_M)$, $U' > 0$; and given that welfare is ordinal, this may be expressed as

$$U = X_R + X_M. \tag{4}$$

Maximization of this utility subject to (1)–(3) entails distributing labor such that

$$L_R + L_M = 1, \text{ and } f_M'(L_M) = f_R'(L_R). \tag{5}$$

Let the amount of labor, output, and utility in this *optimum* condition be written as $[L_R^*, L_M^*, X_R^*, X_M^*, U^*] = E^*$.

The optimum condition E^*, however, is unattainable in an HT economy. In the HT economy the wage paid in the urban sector has an institutional lower bound at $\bar{w}$. Hence,

$$w \geq \bar{w}, \tag{6}$$

where w is the urban market wage and is expressed in units of output. In the HT model $\bar{w}$ is above the laissez-faire equilibrium wage $f_M'(L_M^*)$.

2. This should be treated as a short-run model with fixed capital endowment in each sector.

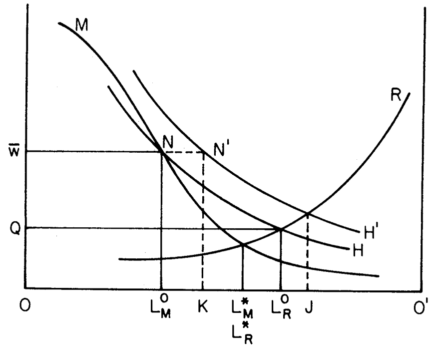

FIGURE I

Hence, w settles at $\overline{w}$. The urban sector maximizes profits, implying that

$$f'_M = \overline{w}. \tag{7}$$

Laborers maximize expected earnings. The rural sector is run along competitive lines, and everybody is provided with employment. The probability of getting a job in the urban sector is given by the rate of urban employment: $L_M/(1 - L_R)$. Hence, migration equilibrium is attained when

$$f'_R = \overline{w}\, L_M/(1 - L_R). \tag{8}$$

Solving (1), (2), (7), and (8), we get the values of L_R, L_M, X_R and X_M. Using (4), we can determine social welfare. These values, denoted by $[L_R^0, L_M^0, X_R^0, X_M^0, U^0] = E^0$, represent the HT equilibrium. Given $\overline{w} > f'_M(L_M^*)$, this equilibrium implies the existence of urban unemployment and sub-optimal production.

A simple diagram can be developed to depict the HT model. In Figure I, let M and R be the marginal product of labor curves with 0 and 0′ as origins, respectively. The optimum condition (i.e., maximum output) involves employing $0L_M^*$ and $0'L_R^*$ in the urban and rural sectors.

Given an institutional wage of $\overline{w}$, the urban sector employs $0L_M^0$ labor. What is the level of rural employment? For that we have to draw a rectangular hyperbola H, through $N = (L_M^0, \overline{w})$. The point of intersection between H and R gives the rural equilibrium. $0'L_R^0$ is the level of rural employment. This is so because $(\overline{w})(0L_M^0) = (0Q)(0L_R^0)$, since H is a rectangular hyperbola. This implies that $(\overline{w})(0L_M^0)/(0L_R^0) = 0Q$. This is the same as (8), and hence L_M^0 and L_R^0 in Figure I represent the HT equilibrium. Urban unemployment consists of $L_M^0\, L_R^0$.

The rectangular hyperbola turns out to be a handy instrument for describing an HT equilibrium. Given any urban wage and employment level, we draw a rectangular hyperbola through the "urban wage-employment" point (N, in the above analysis), and the point of intersection of this rectangular hyperbola with the rural marginal product curve gives us the level of rural employment. The explanation is the same as the one given above.

Harris and Todaro argue that, in their model, the generation of urban employment induces more migration, and consequently urban unemployment may or may not decrease. This is easily seen in Figure I. Let the urban sector employ $L_M^0 K$ more at the fixed wage $\overline{w}$ because of governmental pressure. Then the relevant rectangular hyperbola is the one through N'. This is labeled H' in the diagram. It is obvious that rural employment drops by $L_R^0 J$. Hence urban unemployment, instead of dropping by $L_M^0 K$, drops by $L_M^0 K - L_R^0 J$. This can be positive or negative, and hence unemployment could decrease or increase.

III. Standard Policy Prescriptions

The Harris-Todaro Prescription

In the earlier models of the dual economy, such as that of Marglin [1967], the level of unemployment could be decreased by lowering the shadow wage (i.e., increasing wage subsidy) in the urban sector. In the HT model this need not happen. While a subsidy of S would increase urban employment, that in turn would induce further migration, and net unemployment may rise or fall. However, if the subsidy in the urban sector is raised to $\overline{S}$, where $\overline{S}$ is defined as follows:

$$\overline{S} = \overline{w} - f_M'(1 - \overline{L}_R), \tag{9}$$

where $\overline{L}_R$ is such that $f_R'(\overline{L}_R) = \overline{w}$,[3] then full employment is attained. This has been proved by Srinivasan and Bhagwati [1975].

THEOREM 1. There exists a unique equilibrium corresponding to each wage subsidy S to the manufacturing sector in the interval $[0,\overline{S}]$. At $\overline{S}$ full employment is reached.

The proof of this is obvious, once we analyze this diagrammatically as done below. I shall refer to $\overline{S}$ (as defined in equation (9)) as

3. The existence of such an L_R is guaranteed by introducing the BS assumption that $\lim_{L_R \to 0} f_R'(L_R) = \infty$ and $\lim_{L_M \to 0} f_M'(L_M) = \infty$. This rules out corner solutions.

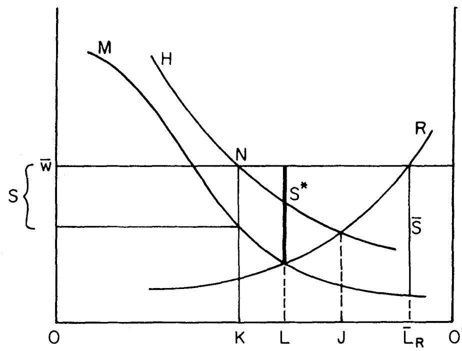

FIGURE II

the HT second-best wage subsidy. Note that this subsidy takes the economy to a full employment situation but not to E^*.

But what if we want to attain the first-best equilibrium E^*? Then Harris and Todaro concluded that we must give a subsidy,

$$S^* = \bar{w} - f'_M(L^*_M), \tag{10}$$

where L^*_M is defined as above, and this, with a complete migration restriction, ensures the attainment of E^*. This led to the well-known HT conclusion that no single policy measure can lead the economy to E^*.

The effect of a subsidy to the urban sector may be analyzed diagrammatically. In Figure II, M is the urban sector's marginal product of labor schedule. Given a subsidy S to the urban sector, the level of urban employment is $0K$. Drawing a rectangular hyperbola through $(K,\bar{w}) = N$, we see that rural employment is $0'J$.

$\bar{L}_R$, defined as that L_R for which $f'_R = \bar{w}$, is shown in Figure II. It is obvious that a subsidy of $\bar{S}$ as defined in equation (9) and depicted in Figure II removes unemployment completely. If a subsidy of S^* is given, then urban employment is at $0L$, and if migration is prohibited, rural employment is at $0'L$. This is the optimum distribution of labor.[4]

The Bhagwati-Srinivasan Prescription

It was Bhagwati and Srinivasan's disagreement with the HT claim that no *single* policy could lead the economy to the optimum, which

4. An alternative way of diagrammatically representing a subsidy is to construct an *effective* marginal product curve by vertically raising the actual marginal product curve by the amount of the subsidy. This method is useful for understanding the effect of large subsidies, e.g., those greater than $\bar{w}$.

led to their 1974 and 1975 papers. They argue that there exists a wage subsidy which, if given to all sectors (i.e., rural and urban) and financed by "some form of non-distortionary taxation," leads to the optimum, i.e., E^*. This optimum subsidy S^* is the same as defined in equation (10); the only difference being that the BS subsidy is for the entire economy, whereas Harris and Todaro suggested it for the urban sector only. I shall refer to a wage subsidy meant for all sectors as a *uniform* wage subsidy. The next theorem is the important contribution of Bhagwati and Srinivasan [1974].

THEOREM 2. There exists a unique equilibrium corresponding to each uniform wage subsidy S in the interval $[0,S^*]$. At S^* the optimum is reached.

The Information Problem

There is, however, a serious information problem with the BS optimal subsidy $S^* = \overline{w} - f'_M(L^*_M)$. While $\overline{w}$ is immediately observable, $f'_M(L^*_M)$ is not. The latter represents the marginal product of labor *when labor is optimally distributed.* Hence, while the BS result is comforting in that it assures us that an optimal subsidy exists, it is not really useful in practice because the value of this optimal subsidy will in general not be known. The same problem arises with the HT second-best subsidy $\overline{S}$. This requires knowledge of $f'_M(1 - \overline{L}_R)$, which again is not an observable.[5]

Fortunately, the situation is not so hopeless as it seems. This is because the BS result can be greatly generalized. While their S^* does lead to E^*, S^* is not the only subsidy that does this. There is a large class of subsidies that lead to optimality.

IV. A Class of Optimal Subsidies

This section proves certain theorems rigorously. Consequently, we need to restate formally some HT conditions in the case where there is a wage subsidy S to both sectors. The urban sector is supposed to maximize profits. This implies two conditions:

$$w > \overline{w} \rightarrow L_R + L_M = 1 \tag{11}$$

$$f'_M(L_M) = w - S. \tag{12}$$

The migration equilibrium condition is now restated as

$$f'_R(L_R) + S = w\, L_M/(1 - L_R). \tag{13}$$

5. This problem is not peculiar to the HT model. All general equilibrium models of dual economies, e.g., Dixit [1971] share this information problem.

While (12) and (13) are familiar conditions (see BS [1974]), (11) needs a word of explanation. It simply says that, given profit maximization, if there is unemployment, then wages cannot be above the institutional minimum. This is obvious enough and is implicitly there in both HT and BS formulations, but there it is not significant because they never consider a case where $w > \bar{w}$. Such cases will arise in our analysis below, and that also accounts for the reason we have w instead of $\bar{w}$ in (12) and (13).

The next theorem generalizes the BS result.

THEOREM 3. (a) If a wage subsidy of $S \geq S^*$ is given to all sectors, then the equilibrium reached is the optimum E^*
(b) This equilibrium is unique.

Proof. (a) An equilibrium is attained if there exist L_R, L_M, and w that satisfy (3), (6), (11), (12), and (13).

Consider that $L_R = L_R^*$ and $L_M = L_M^*$(L_R^* and L_M^* are as defined above). From (5), we know that $L_R^* + L_M^* = 1$. Hence, (11) and (3) are satisfied. From (12), $w = f_M'(L_M^*) + S$. Since $S \geq S^* = \bar{w} - f_M'(L_M^*)$, hence $w \geq \bar{w}$, thereby satisfying (6). From (12), and given that $f_M'(L_M^*) = f_R'(L_R^*)$ (see (5)), we get $f_R'(L_R^*) + S = w$. This can be written as $f_R'(L_R^*) + S = w\, L_M^*/(1 - L_R^*)$, since $L_M^* + L_R^* = 1$. Therefore, L_R^* and L_M^* satisfy (13). Hence $[L_R^*, L_M^*]$ is an equilibrium commensurate with (3), (6), (11), (12), and (13). Given L_R^* and L_M^*, the amounts of output and utility are X_R^*, X_M^*, and U^*. Hence $S \epsilon [S^*, \infty)$ leads to E^*.

(b) Now we have to show that if $S \geq S^*$, then no distribution of labor other than $[L_M^*, L_R^*]$ comprises an equilibrium. Consider a distribution of labor $[\bar{\bar{L}}_M, \bar{\bar{L}}_R]$, such that $\bar{\bar{L}}_M + \bar{\bar{L}}_R \leq 1$ and $\bar{\bar{L}}_M \neq L_M^*$. Now either (i) $\bar{\bar{L}}_R = 1 - \bar{\bar{L}}_M$, or (ii) $\bar{\bar{L}}_R < 1 - \bar{\bar{L}}_M$. If (i), then from (12) and (13), $f_R'(\bar{\bar{L}}_R) = f_M'(\bar{\bar{L}}_M)$. Since $f_R'', f_M'' < 0$, hence $f_R' = f_M'$ and $L_R + L_M = 1$ can be true at some unique L_R and L_M. Since $f_R'(L_R^*) = f_M'(L_M^*)$, we have a contradiction. If (ii), then $\bar{\bar{L}}_M/(1 - \bar{\bar{L}}_R) < 1$ and by (12) and (13) $f_M'(\bar{\bar{L}}_M) > f_R'(\bar{\bar{L}}_R)$. This, which can easily be checked, implies that $f_M'(\bar{\bar{L}}_M) > f_M'(L_M^*)$. Hence, $w = f_M'(L_M) + S > f_M'(L_M^*) + S \geq \bar{w}$, since $S \geq S^*$. This implies that $w > \bar{w}$ and $L_R + \bar{\bar{L}}_M < 1$. This contradicts (11). Hence $\bar{\bar{L}}_M = L_M^*$. It is easily seen that this implies $\bar{\bar{L}}_R = L_R^*$.

This theorem shows that, for all subsidies equal to or greater than S^*, we obtain E^*. If S is raised above S^*, then the only variable that changes is w. In fact, w is a function of S. For all $S \geq S^*$, $w = f_M'(L_M^*) + S$. If $S \leq S^*$, then $w = \bar{w}$. ‖

This theorem implies that we no longer need to know the exact value of S^*. The government can always make sure of reaching E^*, by choosing a sufficiently large S. The next theorem shows that the government's problem is actually even simpler.

THEOREM 4. If a wage subsidy of $S = \bar{w}$ is given to all sectors, then E^* is reached.

Proof. By definition, $S^* = \bar{w} - f'_M(L^*_M)$. Hence, $\bar{w} > S^*$, since by assumption $f'_M > 0$, for all L_M. Consequently, by Theorem 3, a subsidy of $\bar{w}$ guarantees that we reach E^*. ||

Thus, the government need not give an arbitrarily high subsidy to make sure of reaching E^*. A subsidy equal to $\bar{w}$, which is an observable parameter of the economy, suffices. The problem of having information on variables at the optimal situation is, therefore, totally bypassed.

From Theorem 3 we know that for all uniform subsidies above S^*, social welfare remains at the maximum. What happens to social welfare for subsidies in the range 0 to S^*? Srinivasan and Bhagwati [1975] studied the effects of an *urban* wage subsidy on social welfare, and concluded that U does not always increase as the urban wage subsidy S increases. Indeed, U may even fall over certain ranges. Fortunately, for a *uniform* subsidy, monotonicity always holds.

THEOREM 5. If S is a wage subsidy given to both sectors and $S \epsilon [0,S^*]$, then as S increases, social welfare U also increases.

Proof. $U = X_M + X_R = f_M(L_M) + f_R(L_R)$.

It is clear from (11), (12), and (13) that L_M and L_R depend on S.

Hence,

$$\frac{dU}{dS} = f'_M \frac{dL_M}{dS} + f'_R \frac{dL_R}{dS}. \tag{14}$$

From (12),

$$f''_M \frac{dL_M}{dS} = -1; \quad \text{i.e.,} \quad \frac{dL_M}{dS} = \frac{-1}{f''_M}.$$

From (13),

$$f'_R + S - L_R f'_R - SL_R - \bar{w}L_M = 0.$$

By differentiating with respect to S:

$$f''_R \frac{dL_R}{dS} + 1 - L_R f''_R \frac{dL_R}{dS} - \frac{dL_R}{dS} f'_R - S\frac{dL_R}{dS} - L_R - \bar{w}\frac{dL_M}{dS} = 0.$$

By grouping and substituting for dL_M/dS, we get

$$\frac{dL_R}{dS} = \frac{\{L_R - 1 - \bar{w}/f''_M\}}{\{(1 - L_R) f''_R - f'_R - S\}}.$$

By substituting for dL_M/dS and dL_R/dS into (14), we get

$$\frac{dU}{dS} = -\frac{f'_M}{f''_M} + \frac{f'_R\{L_R - 1 - \overline{w}/f''_M\}}{(1 - L_R)f''_R - f'_R - S}$$

$$= \frac{-f'_M(1 - L_R)f''_R + L_R f'_R f''_M - f'_R f''_M + f'_R(f'_M - \overline{w}) + Sf'_M}{f''_M\{(1 - L_R)f''_R - f'_R - S\}}$$

$$= \frac{(L_R - 1)f''_R f'_M + (L_R - 1)f'_R f''_M + S(f'_M - f'_R)}{f''_M\{(1 - L_R)f''_R - f'_R - S\}},$$

since

$$f'_M - \overline{w} = -S,$$

by (12). Noting that $L_R + L_M \leq 1, f'_i > 0, f''_i < 0, i \in \{R,M\}$, and $f'_M > f'_R$ (this is easily checked), we know that $dU/dS > 0$. ||

Results, similar to those derived for uniform subsidies above, hold for the HT type sector-specific subsidies. Theorem 1 states that a wage subsidy of $\overline{S}$ to the urban sector removes unemployment. It can be shown that for all urban wage subsidy $S \in [\overline{S}, \infty)$, unemployment is removed. This result, however, is not so useful as Theorem 3 because it is easily checked that as S rises above $\overline{S}$, social welfare U, drops.

In conclusion, we may recall that the BS and HT models assume that the subsidy is financed by "some form of nondistortionary tax." This problem demands further investigation. When doling out large economy-wide subsidies, there is always the danger of inflation and of the savings rate dropping. In fact, research into subsidy finance may reveal that no completely "nondistortionary tax" exists, and in reality we have to be reconciled to a smaller "second-best" subsidy.

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REFERENCES

Bhagwati, J. N., and T. N. Srinivasan, "On Reanalyzing the Harris-Todaro Model: Policy Rankings in the case of Sector-Specific Sticky Wages," *American Economic Review,* LXIV (1974), 502–08.

Dixit, A., "Short-Run Equilibrium and Shadow Prices in the Dual Economy," *Oxford Economic Papers,* XXIII (1971), 384–400.

Fields, G., "Rural-Urban Migration, Urban Unemployment and Under-Employment, and Job-search Activity in LDCs," *Journal of Development Economics,* II (1975), 165–87.

Harris, J. R., and M. P. Todaro, "Migration, Unemployment and Development: A Two-Sector Analysis," *American Economic Review,* LX (1970), 126–42.

Lewis, W. A., "Economic Development with Unlimited Supplies of Labour," *Manchester School of Economic and Social Studies,* XXII (1954), 139–91.

Marglin, S. A., *Public Investment Criteria* (London: Unwin, 1967).

Mazumdar, D., "The Rural-Urban Wage Gap, Migration and the Shadow Wage," *Oxford Economic Papers,* XXVIII (1976), 406–25.
Sen, A. K., *Choice of Techniques,* 3rd ed. (Oxford: Blackwell, 1968).
Srinivasan, T. N., and J. N. Bhagwati, "Alternative Policy Rankings in a Large, Open Economy with Sector-Specific, Minimum Wages," *Journal of Economic Theory,* XI (1975), 356–71.
Todaro, M. P., "A Model of Labor Migration and Urban Unemployment in Less Developed Countries," *American Economic Review,"* LIX (1969), 138–48.